Math 202 Test 3B

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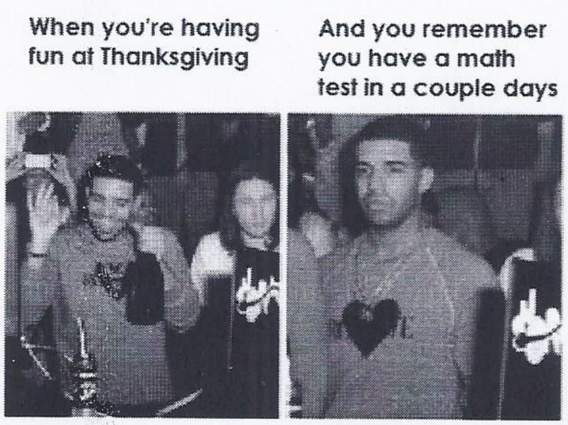
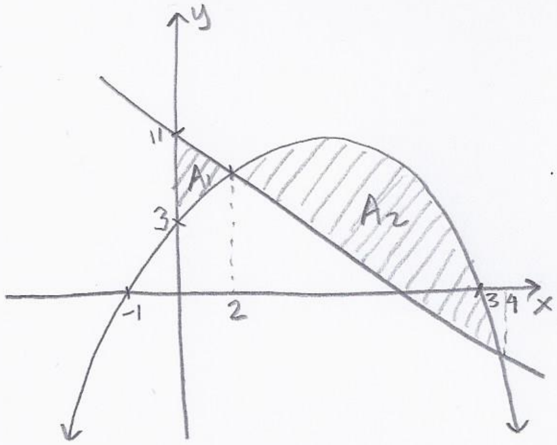
Note that both sides of each page may have printed material.

Instructions:

1. Read the instructions.
2. Panic!!! Kidding, don't panic! I repeat, do NOT panic!
3. Complete all problems in the actual test. Bonus problems are, of course, optional.
4. Show ALL your work to receive full credit. You will get 0 credit for simply writing down the answers (unless it's a case of fill in the blank or state a definition, etc.)
5. Write neatly so that I am able to follow your sequence of steps and box your answers.
6. Read through the exam and complete the problems that are easy (for you) first!
7. No scrap paper, calculators, notes or other outside aids allowed—including divine intervention, telepathy, knowledge osmosis, the smart kid that may be sitting beside you or that friend you might be thinking of texting.
8. In fact, **cell phones should be out of sight!**
9. Use the correct notation and write what you mean! x^2 and $x2$ are not the same thing, for example, and I will grade accordingly.
10. Do NOT commit any of the blasphemies or mistakes I mentioned in the syllabus. I will actually mete out punishment in the way I said I would. I wasn't kidding.
11. Other than that, have fun and good luck!



1. (25 points) Find the area bounded between the curves $y = 3 + 2x - x^2$ and $y = -4x + 11$ for $0 \leq x \leq 4$. Include a sketch of the region and shade the required area.



$$A = A_1 + A_2$$

$$= \int_0^2 -4x+11-(3+2x-x^2)\,dx + \int_2^4 3+2x-x^2-(-4x+11)\,dx$$

$$= \int_0^2 8-6x+x^2\,dx - \int_2^4 8-6x+x^2\,dx$$

$$= 8x-3x^2+\frac{x^3}{3}\Big|_0^2 - \left(8x-3x^2+\frac{x^3}{3}\right)\Big|_2^4$$

$$= 16-12+\frac{8}{3}-0-\left(32-48+\frac{64}{3}-\left(16-12+\frac{8}{3}\right)\right)$$

$$= 4+\frac{8}{3}+16-\frac{64}{3}+4+\frac{8}{3}$$

$$= 24-\frac{48}{3}$$

$$= \boxed{8}$$

Intersection points

$3+2x-x^2 = -4x+11$

$\Rightarrow x^2-6x+8=0$

$\Rightarrow (x-4)(x-2)=0$

$\Rightarrow x=4,\ x=2$

For $y=3+2x-x^2$

x-int: $-(x-3)(x+1)=0$

$\Rightarrow x=3,\ x=-1$

y-int: $y=3$

For $y=-4x+11$

x-int: $x=11/4$

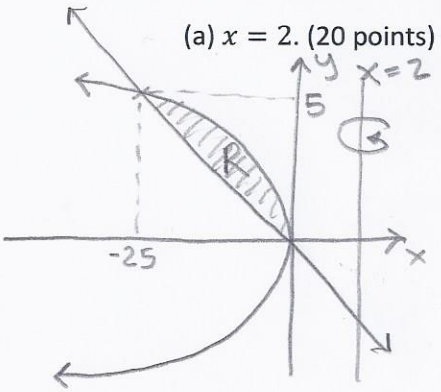
y-int: $y=11$

5 pts for diagram
5 pts for intersection calculations
5 pts for correct integral set up
5 pts to get to fifth to last line
4 pts to plug in numbers correctly
1 pt for correct final answer

2. Let R be the region bounded by $x = -y^2$ and $y = -\frac{x}{5}$. Find the volume of the solid obtained by rotating R about the line:

- 5 pts for sketch
- 5 pts for finding intersections
- 5 pts for setting up integral
- 5 pts for evaluating the integral

(a) $x = 2$. (20 points)



Intersections

$-y^2 = -5y$

$\Rightarrow y^2 - 5y = 0$

$y = 0,\ y = 5$

$\downarrow \qquad \downarrow$

$x = 0 \qquad x = -25$

Using Shell Method

$r = 2 - x$

$h = \sqrt{-x} + \frac{x}{5}$

$$V = 2\pi\int_{-25}^{0} (2-x)\left((-x)^{1/2} + \tfrac{x}{5}\right)dx$$

$$= 2\pi\int_{-25}^{0} 2(-x)^{1/2} + \tfrac{2}{5}x - (-x)^{3/2} - \tfrac{x^2}{5}\,dx$$

$$= 2\pi\left[-\tfrac{4}{3}(-x)^{3/2} + \tfrac{x^2}{5} - \tfrac{2}{5}(-x)^{5/2} - \tfrac{x^3}{15}\right]_{-25}^{0}$$

$$= -2\pi\left(-\tfrac{500}{3} + 125 - 1250 + \tfrac{3125}{3}\right)$$

$$= \boxed{500\pi}$$

Full credit for getting to this point

Using Disk/Washer Method

$R = 2 + 5y, \quad r = 2 + y^2$

$$\Rightarrow V = \pi\int_0^5 (2+5y)^2 - (2+y^2)^2\,dy$$

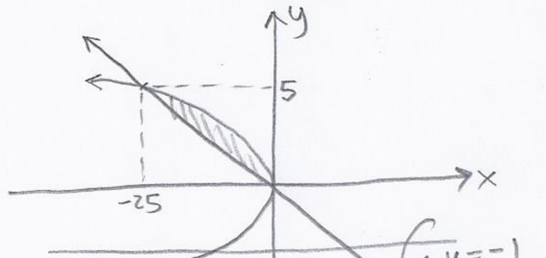
$$= \pi\int_0^5 20y + 21y^2 - y^4\,dy$$

$$= \pi\left[10y^2 + \tfrac{21}{3}y^3 - \tfrac{y^5}{5}\right]_0^5$$

$$= \pi[250 + 875 - 625]$$

$$= \boxed{500\pi}$$

(b) $y = -1$. (20 points)



Full credit for getting to this point

Using Disk/Washer Method

$R = (-x)^{1/2} + 1, \quad r = -\frac{x}{5} + 1$

$$V = \pi\int_{-25}^{0} \left((-x)^{1/2}+1\right)^2 - \left(-\tfrac{x}{5}+1\right)^2 dx$$

$$= \pi\int_{-25}^{0} -\tfrac{3x}{5} + 2(-x)^{1/2} - \tfrac{x^2}{25}\,dx$$

$$= \pi\left[-\tfrac{3x^2}{10} - \tfrac{4}{3}(-x)^{3/2} - \tfrac{x^3}{75}\right]_{-25}^{0}$$

$$= -\pi\left[-\tfrac{3(25)^2}{10} - \tfrac{4}{3}(25)^{3/2} - \tfrac{(-25)^3}{75}\right]$$

$$= \boxed{\tfrac{875\pi}{6}}$$

Using Shell Method

$r = y + 1, \quad h = -y^2 + 5y$

$$V = 2\pi\int_0^5 (y+1)(-y^2+5y)\,dy$$

$$= 2\pi\int_0^5 -y^3 + 4y^2 + 5y\,dy$$

$$= 2\pi\left[-\tfrac{y^4}{4} + \tfrac{4}{3}y^3 + \tfrac{5}{2}y^2\right]_0^5$$

$$= 2\pi\left[-\tfrac{5^4}{4} + \tfrac{4}{3}(5)^3 + \tfrac{5}{2}(5)^2\right]$$

$$= \boxed{\tfrac{875\pi}{6}}$$