Math 202 Test 2A
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Note that both sides of each page may have printed material.

Instructions:

1. Read the instructions.

2. Panic!!! Kidding, don't panic! I repeat, do NOT panic!

3. Complete all problems in the actual test. Bonus problems are, of course, optional.

4. Show ALL your work to receive full credit. You will get 0 credit for simply writing down the answers (unless it's a case of fill in the blank or state a definition, etc.)

5. Write neatly so that I am able to follow your sequence of steps and box your answers.

6. Read through the exam and complete the problems that are easy (for you) first!

7. No scrap paper, calculators, notes or other outside aids allowed—including divine intervention, telepathy, knowledge osmosis, the smart kid that may be sitting beside you or that friend you might be thinking of texting.

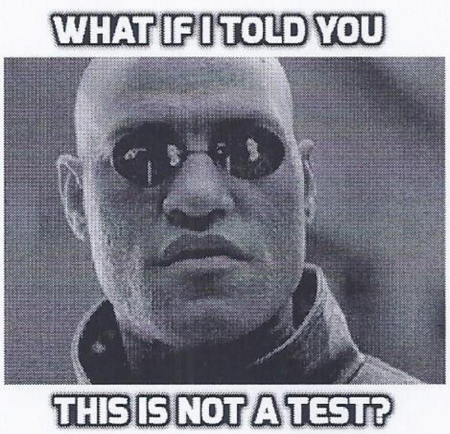
8. In fact, **cell phones should be out of sight!**

9. Use the correct notation and write what you mean! x^2 and $x2$ are not the same thing, for example, and I will grade accordingly.

10. Do NOT commit any of the blasphemies or mistakes I mentioned in the syllabus. I will actually mete out punishment in the way I said I would. I wasn't kidding.

11. Other than that, have fun and good luck!

Remember: Avoid confusion



1. Compute the following integrals (10 points each):

(a) $\int 2x^3 \sin x \; dx$

+	$6x^2$	$-\cos x$	$\to -2x^3 \cos x$
−	$12x$	$-\sin x$	$\to 6x^2 \sin x$
+	12	$\cos x$	$\to 12x \cos x$
−	0	$\sin x$	$\to -12 \sin x$

$$\int 2x^3 \sin x \, dx = \boxed{-2x^3 \cos x + 6x^2 \sin x + 12x \cos x - 12 \sin x + C}$$

(b) $\int \sqrt{\tan x}\, \sec^4 x \; dx$

$u = \tan x$

$du = \sec^2 x \, dx$

$$\int u^{1/2}(1+u^2)\, du$$

$$= \int \left(u^{1/2} + u^{5/2}\right) du$$

$$= \frac{2}{3} u^{3/2} + \frac{2}{7} u^{7/2} + C$$

$$= \boxed{\frac{2}{3}(\tan x)^{3/2} + \frac{2}{7}(\tan x)^{7/2} + C}$$

(c) $\int_1^e x^{3/2} \ln x \, dx$

$$u = \ln x \qquad dv = x^{3/2}\,dx$$
$$du = \frac{1}{x}\,dx \qquad v = \frac{2}{5}x^{5/2}$$

$$\int_1^e x^{3/2} \ln x \, dx = \frac{2}{5}x^{5/2}\ln x \Big|_1^e - \frac{2}{5}\int_1^e x^{3/2}\,dx$$
$$= \frac{2}{5}x^{5/2}\ln x - \frac{4}{25}x^{5/2}\Big|_1^e$$
$$= \frac{2}{5}e^{5/2} - \frac{4}{25}e^{5/2} + \frac{4}{25}$$
$$= \boxed{\frac{6}{25}e^{5/2} + \frac{4}{25}}$$

(d) $\int \frac{x^2+2x-14}{x^2+2x-8}\,dx$

$$= \int \frac{x^2+2x-8-6}{x^2+2x-8}\,dx$$
$$= \int \left[1 - \frac{6}{(x+4)(x-2)}\right]dx$$
$$= \int \left(1 + \frac{1}{x+4} - \frac{1}{x-2}\right)dx$$
$$= \boxed{x + \ln|x+4| - \ln|x-2| + C}$$

or

$$= \boxed{x + \ln\left|\frac{x+4}{x-2}\right| + C}$$

(e) $\int \frac{\pi x^2}{\sqrt{16-x^2}}\,dx$

$x = 4\sin\theta$

$dx = 4\cos\theta\, d\theta$

$= \pi \int \frac{16\sin^2\theta \cdot \cancel{4\cos\theta}\, d\theta}{\cancel{4\cos\theta}}$

$= 8\pi \int (1 - \cos 2\theta)\, d\theta$

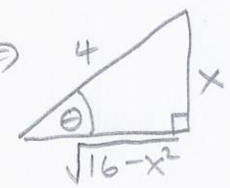
$= 8\pi \left[\theta - \frac{1}{2}\sin 2\theta\right] + C$

$= 8\pi \left[\theta - \sin\theta\cos\theta\right] + C$

$= \boxed{8\pi\left(\sin^{-1}\frac{x}{4} - \frac{x}{16}\sqrt{16-x^2}\right) + C}$

$\sin\theta = \frac{x}{4}$

$\Rightarrow$



$\Rightarrow \cos\theta = \frac{1}{4}\sqrt{16-x^2}$

$\Rightarrow \theta = \sin^{-1}\frac{x}{4}$

2. Write down the partial fractions decomposition of the following. Do not attempt to solve for the arbitrary constants (2 points each):

(a) $\frac{3x^2+1}{x^3(x-1)(x^2+4)^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x-1} + \frac{Ex+F}{x^2+4} + \frac{Gx+H}{(x^2+4)^2}$

(b) $\frac{5\cancel{x}}{x^{\cancel{2}}(x^2+1)^2(x^2-9)} = \frac{A}{x} + \frac{Bx+C}{x^2+1} + \frac{Dx+E}{(x^2+1)^2} + \frac{F}{x-3} + \frac{G}{x+3}$

(c) $\frac{x^2+1}{x^2-1} = 1 + \frac{2}{x^2-1} = 1 + \frac{A}{x-1} + \frac{B}{x+1}$

(d) $\frac{\pi - ex^3}{x^6+x^3} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x+1} + \frac{Ex+F}{x^2-x+1}$

$x^3(x^3+1)$

$\cancel{x}(x^2+\cancel{2x+1})$

(e) $\frac{x^3+2x^2+x}{x^{\cancel{2}}(x^2+2x+1)(x+2)} = \frac{A}{x} + \frac{B}{x+2}$

3. Set up (you should leave your answer as a sum of fractions) the approximations for

$$\int_{-2}^{6} \frac{1}{2x+7}\,dx$$

using $n = 4$ and:

-2 0 2 4 6

x_0 x_1 x_2 x_3 x_4

(a) The Trapezoidal rule (5 points):

$$\Delta x = \frac{6-(-2)}{4} = 2$$

$$T_4 = \frac{\Delta x}{2}\left(f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4)\right)$$

$$= 1\cdot\left(\frac{1}{-4+7} + \frac{2}{7} + \frac{2}{4+7} + \frac{2}{8+7} + \frac{1}{12+7}\right)$$

$$= \boxed{\frac{1}{3} + \frac{2}{7} + \frac{2}{11} + \frac{2}{15} + \frac{1}{19}}$$

(b) Simpson's rule (5 points):

$$S_4 = \frac{\Delta x}{3}\left(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)\right)$$

$$= \frac{2}{3}\left(\frac{1}{-4+7} + \frac{4}{7} + \frac{2}{4+7} + \frac{4}{8+7} + \frac{1}{12+7}\right)$$

$$= \boxed{\frac{2}{3}\left(\frac{1}{3} + \frac{4}{7} + \frac{2}{11} + \frac{4}{15} + \frac{1}{19}\right)}$$

(c) What is the error in the Trapezoidal rule approximation here? (5 points)

$$f = (2x+7)^{-1}$$
$$\Rightarrow f' = -2(2x+7)^{-2}$$
$$\Rightarrow f'' = 8(2x+7)^{-3}$$
$$\Rightarrow |f''| = \left|\frac{8}{(2x+7)^3}\right| \le \frac{8}{27} \text{ on } [-2,6]$$
$$\Rightarrow K = 8/27$$

$$E_T = \frac{K(b-a)^3}{12n^2}$$

$$= \boxed{\frac{\frac{8}{27}(8)^3}{12(4)^2}}$$

(d) What must n be to ensure the Trapezoidal approximation is accurate to at least 0.01? (5 points)

We want $|E_T| \le 0.01$

$$\Rightarrow \frac{\frac{8}{27}(8)^3}{12n^2} \le \frac{1}{100}$$

$$\Rightarrow \frac{12n^2}{\frac{8^4}{27}} \ge 100$$

$$\Rightarrow n \ge \sqrt{\frac{100(8^4)}{27\cdot 12}} \longrightarrow$$

n must be the smallest integer greater than this number