Math 202 Test 1B
October 6, 2015

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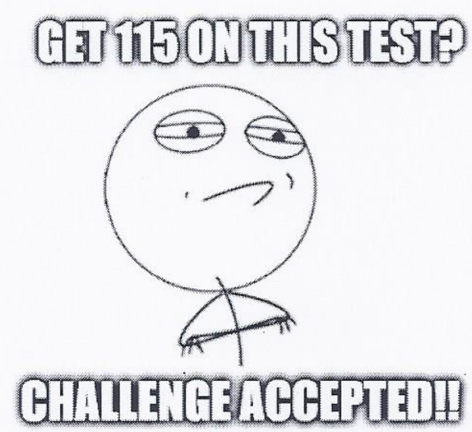
Note that both sides of each page may have printed material.

Instructions:

1. Read the instructions.
2. Panic!!! Kidding, don't panic! I repeat, do NOT panic!
3. Complete all problems in the actual test. Bonus problems are, of course, optional.
4. Show ALL your work to receive full credit. You will get 0 credit for simply writing down the answers (unless it's a case of fill in the blank or state a definition, etc.)
5. Write neatly so that I am able to follow your sequence of steps and box your answers.
6. Read through the exam and complete the problems that are easy (for you) first!
7. No scrap paper, calculators, notes or other outside aids allowed—including divine intervention, telepathy, knowledge osmosis, the smart kid that may be sitting beside you or that friend you might be thinking of texting.
8. In fact, **cell phones should be out of sight!**
9. Use the correct notation and write what you mean! x^2 and $x2$ are not the same thing, for example, and I will grade accordingly.
10. Do NOT commit any of the blasphemies or mistakes I mentioned in the syllabus. I will actually mete out punishment in the way I said I would. I wasn't kidding.
11. Other than that, have fun and good luck!



1. Find y' for the following (10 points each):

(a) $y = x^{x^3} + 2^{\cos x}$

$$\Rightarrow y = e^{x^3 \ln x} + 2^{\cos x}$$

$$\Rightarrow y' = (3x^2 \ln x + x^2) x^{x^3} - \sin x \cdot 2^{\cos x} \cdot \ln 2$$

$$= \boxed{(3\ln x + 1) x^{x^3+2} - \sin x \cdot 2^{\cos x} \cdot \ln 2}$$

OR Set $g = x^{x^3}$

$$\Rightarrow \ln g = x^3 \ln x$$

$$\Rightarrow \frac{g'}{g} = 3x^2 \ln x + x^2$$

$$\Rightarrow g' = g(3x^2 \ln x + x^2)$$

$$= x^{x^3}(3x^2 \ln x + x^2)$$

$$= x^{x^3+2}(3\ln x + 1)$$

$$\Rightarrow y' = g' + \frac{d}{dx} 2^{\cos x}$$

$$= \boxed{(3\ln x + 1) x^{x^3+2} - \sin x \cdot 2^{\cos x} \cdot \ln 2}$$

(b) $y = \tan^{-1}(e^{x^3}) + x^3 \ln \cos x$

$$y' = \frac{3x^2 e^{x^3}}{1 + (e^{x^3})^2} + 3x^2 \ln \cos x + x^3 \cdot \frac{-\sin x}{\cos x}$$

$$= \boxed{\frac{3x^2 e^{x^3}}{1 + e^{2x^3}} + 3x^2 \ln \cos x - x^3 \tan x}$$

(c) $\ln\sqrt{x+y} = x^3 + e^{\pi^3} + \cos(x+y)$

$$\Rightarrow \frac{1}{2}\ln(x+y) = x^3 + e^{\pi^3} + \cos(x+y)$$

$$\Rightarrow \frac{1}{2}\cdot\frac{1+y'}{x+y} = 3x^2 + (1+y')\cdot -\sin(x+y)$$

$$\Rightarrow \frac{1}{2(x+y)} + \frac{y'}{2(x+y)} = 3x^2 - \sin(x+y) - y'\sin(x+y)$$

$$\Rightarrow y'\left(\frac{1}{2(x+y)} + \sin(x+y)\right) = 3x^2 - \sin(x+y) - \frac{1}{2(x+y)}$$

$$\Rightarrow y' = \frac{3x^2 - \sin(x+y) - \frac{1}{2(x+y)}}{\frac{1}{2(x+y)} + \sin(x+y)}$$

$$= \boxed{\frac{6x^2(x+y) - 2(x+y)\sin(x+y) - 1}{1 + 2(x+y)\sin(x+y)}}$$

2. Evaluate the following integrals (10 points each):

(a) $\int \frac{\sin(5\sqrt{x})}{3\sqrt{x}}\,dx$

$u = 5\sqrt{x}$

$du = \frac{5}{2\sqrt{x}}\,dx$

$\Rightarrow \frac{2}{5}\,du = \frac{1}{\sqrt{x}}\,dx$

$\Rightarrow \frac{2}{15}\int \sin u\,du$

$= -\frac{2}{15}\cos u + C$

$= \boxed{-\frac{2}{15}\cos(5\sqrt{x}) + C}$

changing limits

(b) $\displaystyle\int_1^e \frac{\sqrt[5]{\ln x}}{3x}\,dx$

$u^5 = \ln x$

when $x = e$, $u = 1$

when $x = 1$, $u = 0$

$\Rightarrow 5u^4\,du = \frac{1}{x}\,dx$

$\Rightarrow \frac{5}{3}\int_0^1 u \cdot u^4\,du$

$= \frac{5}{3}\left.\frac{u^6}{6}\right|_0^1$

$= \frac{5}{3}\cdot\frac{1}{6}$

$= \boxed{\frac{5}{18}}$

OR

$u = \ln x$

$du = \frac{1}{x}\,dx$

$\Rightarrow \frac{1}{3}\int_0^1 \sqrt[5]{u}\,du$

$= \frac{1}{3}\int_0^1 u^{1/5}\,du$

$= \left.\frac{1}{3}\cdot\frac{5}{6}u^{6/5}\right|_0^1$

$= \boxed{\frac{5}{18}}$

You could also back substitute for u and use the old limits instead of changing the limits.

(c) $\displaystyle\int \frac{\pi^e x^2}{\sqrt[3]{5-3x^3}}\,dx$

$u = 5 - 3x^3$

$du = -9x^2\,dx$

$\Rightarrow -\frac{1}{9}\,du = x^2\,dx$

$\Rightarrow -\frac{\pi^e}{9}\int u^{-1/3}\,du$

$= -\frac{\pi^e}{\cancel{3}\cancel{9}}\,\frac{\cancel{3}}{2}u^{2/3} + C$

$= -\frac{\pi^e}{6}u^{2/3} + C$

$= \boxed{-\frac{\pi^e}{6}(5-3x^3)^{2/3} + C}$

OR

$u^3 = 5 - 3x^3$

$\Rightarrow 3u^2\,du = -9x^2\,dx$

$\Rightarrow -\frac{u^2}{3}\,du = x^2\,dx$

$\Rightarrow -\frac{\pi^e}{3}\int \frac{u^{\cancel{2}}}{\cancel{u}}\,du$

$= -\frac{\pi^e}{3}\cdot\frac{u^2}{2} + C$

$= \boxed{-\frac{\pi^e}{6}(5-3x^3)^{2/3} + C}$

3. Compute the following limits (10 points each):

(a) $\lim_{x\to -1} \frac{x^2-x-2}{x+1} = \lim_{x\to -1} \frac{(x-2)\cancel{(x+1)}}{\cancel{x+1}}$

$= \boxed{-3}$

(b) $\lim_{x\to 0} \frac{e^x-(1-x)}{x^4} \longrightarrow \frac{0}{0}$

Apply L'Hôpital's

$\overset{L'H}{=} \lim_{x\to 0} \frac{e^x+1}{4x^3} \longrightarrow \frac{2}{0} \longrightarrow$ can't apply L'H!

$\lim_{x\to 0^-} \frac{e^x+1}{4x^3} = -\infty$ while $\lim_{x\to 0^+} \frac{e^x+1}{4x^3} = +\infty$

$\Rightarrow$ $\boxed{\text{Limit D.N.E!}}$