MATH 202 Quiz 9 – Version B

November 3, 2015

Name: ANSWERS

Instructions: No calculators! Use your own scrap paper and write your answers in the space provided.

1. For the given integral, use $n = 4$ to approximate it using the indicated technique:

$$\int_0^{\pi^2} \cos\sqrt{x}\, dx$$

(a) Trapezoidal rule approximation: $\frac{\pi^2}{8}\left(1 + 2\cos\frac{\pi}{2} + 2\cos\frac{\pi}{\sqrt{2}} + 2\cos\frac{\sqrt{3}\pi}{2} - 1\right)$

(b) Simpson's rule approximation: $\frac{\pi^2}{12}\left(1 + 4\cos\frac{\pi}{2} + 2\cos\frac{\pi}{\sqrt{2}} + 4\cos\frac{\sqrt{3}\pi}{2} - 1\right)$

(c) Midpoint rule approximation: $\frac{\pi^2}{4}\left(\cos\frac{\pi}{\sqrt{8}} + \cos\sqrt{\frac{3}{8}}\pi + \cos\sqrt{\frac{5}{8}}\pi + \cos\sqrt{\frac{7}{8}}\pi\right)$

2. For the above approximations, what is the error associated with:

(a) The Trapezoidal rule: $E_T =$ $K\pi^6/12n^2$

(b) The Midpoint rule: $E_M =$ $K\pi^6/24n^2$

$K = 1/2$ works, but this is difficult to determine

3. How large must n be to have an approximation to with 0.01 accuracy using the Trapezoidal rule?

$n > \sqrt{\frac{100K\pi^6}{12}} = \sqrt{\frac{25K\pi^6}{3}} = 5\pi^3\sqrt{\frac{K}{3}}$

4. If the integral below converges, find its value. If it diverges, state so:

(a) $\int_{-\infty}^{2} \frac{x^2}{4+x^6}\, dx =$ $\pi/6$

(b) $\int_0^1 \frac{\ln x}{\sqrt{x}}\, dx =$ -4

(c) $\int_0^{\infty} x^3 \ln x\, dx =$ $4\ln 2 - 1$

(d) $\int_0^{\infty} x^3 e^{-x^4}\, dx =$ $1/4$

5. Use the comparison theorem to determine if the integral converges or diverges:

(a) $\int_1^{\infty} \frac{x+2}{\sqrt{x^4-2x}}\, dx$

This integral converges/diverges (circle one). I compared it to: $\int_1^{\infty} \frac{x+2}{x^2}\, dx$ (state an integral)

(b) $\int_0^{\infty} \frac{\arctan x}{3+e^x}\, dx$

This integral converges/diverges (circle one). I compared it to: $\int_0^{\infty} \frac{\pi/2}{e^x}\, dx$ (state an integral)

Bonus:

1. Compute the area between the two given curves:

(a) $y = x$ and $y = x^2$.

Integral set up: $\int_0^1 x - x^2\, dx$ Area: $1/6$