MATH 202 Quiz 9 – Version A

November 3, 2015

Name: ANSWERS

Instructions: No calculators! Use your own scrap paper and write your answers in the space provided.

1. For the given integral, use $n = 4$ to approximate it using the indicated technique:

$$\int_0^{\pi^2} \sin\sqrt{x}\, dx$$

$\sin 0$ $\quad 2 \quad$ $\sin\pi$

(a) Trapezoidal rule approximation: $\frac{\pi^2}{8}\left(0 + 2\sin\frac{\pi}{2} + 2\sin\frac{\pi}{\sqrt{2}} + 2\sin\frac{\sqrt{3}\pi}{2} + 0\right)$

(b) Simpson's rule approximation: $\frac{\pi^2}{12}\left(0 + 4\sin\frac{\pi}{2} + 2\sin\frac{\pi}{\sqrt{2}} + 4\sin\frac{\sqrt{3}\pi}{2} + 0\right)$

(c) Midpoint rule approximation: $\frac{\pi^2}{4}\left(\sin\frac{\pi}{\sqrt{8}} + \sin\sqrt{\frac{3}{8}}\pi + \sin\sqrt{\frac{5}{8}}\pi + \sin\sqrt{\frac{7}{8}}\pi\right)$

K can get arbitrarily large for the given interval

2. For the above approximations, what is the error associated with:

(a) The Trapezoidal rule: $E_T =$ $K\pi^6/12n^2$

(b) The Midpoint rule: $E_M =$ $K\pi^6/24n^2$

3. How large must n be to have an approximation to with 0.01 accuracy using the Trapezoidal rule?

$n >$ $\sqrt{\frac{100K\pi^6}{12}} = \sqrt{\frac{25K\pi^6}{3}} = 5\pi^3\sqrt{\frac{K}{3}}$

4. If the integral below converges, find its value. If it diverges, state so:

(a) $\displaystyle\int_{-\infty}^{\infty} \frac{x^2}{9 + x^6}\, dx =$ $\frac{\pi}{9}$

(b) $\displaystyle\int_0^1 \frac{\ln x}{\sqrt{x}}\, dx =$ -4

(c) $\displaystyle\int_0^2 x^2 \ln x\, dx =$ $\frac{8}{3}\ln 2 - \frac{8}{9}$

(d) $\displaystyle\int_0^{\infty} x^2 e^{-x^3}\, dx =$ $1/3$

5. Use the comparison theorem to determine if the integral converges or diverges:

(a) $\displaystyle\int_0^{\infty} \frac{\arctan x}{2 + e^x}\, dx$

This integral converges/diverges (circle one). I compared it to: $\displaystyle\int_0^{\infty} \frac{\pi/2}{e^x}\, dx$ (state an integral)

(b) $\displaystyle\int_1^{\infty} \frac{x + 1}{\sqrt{x^4 - x}}\, dx$

This integral converges/diverges (circle one). I compared it to: $\displaystyle\int_1^{\infty} \frac{x+1}{x^2}\, dx$ (state an integral)

Bonus:

1. Compute the area between the two given curves:

(a) $y = \sqrt{x}$ and $y = x^2$.

Integral set up: $\displaystyle\int_0^1 \sqrt{x} - x^2\, dx$ Area: $1/3$