

MATH 202 Quiz 7 – Version B

October 20, 2015

Name: ANSWERS

Instructions: No calculators! Use your own scrap paper and write your answers in the space provided.

1. Simplify or perform the long division:

(a) $\frac{x^3-5}{x+1} = x^2 - x + 1 - \frac{6}{x+1}$ (b) $\frac{x^2+5x+1}{x^2+2} = 1 + \frac{5x-1}{x^2+2}$

2. Write down the partial fraction decomposition of the following. Do NOT solve for the arbitrary constants:

(a) $\frac{2x^2-7}{x^3(x-2)(x^2+3)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x-2} + \frac{Ex+F}{x^2+3}$

(b) $\frac{4-3x^2}{(x^2+4x+4)(x+3)} = \frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{C}{x+3}$

(c) $\frac{7}{\frac{x^6-x^3}{x^2(x^2-1)}} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x-1} + \frac{Ex+F}{x^2+x+1}$

3. Complete the following table of trig substitutions:

| Expression | Substitution | Identity |
|--------------------|--|--|
| $\sqrt{a^2 - x^2}$ | $x = a \sin \theta$ or $x = a \cos \theta$ | $1 - \sin^2 \theta = \cos^2 \theta$ or $1 - \cos^2 \theta = \sin^2 \theta$ |
| $\sqrt{x^2 + a^2}$ | $x = a \tan \theta$ | $\tan^2 \theta + 1 = \sec^2 \theta$ |
| $\sqrt{x^2 - a^2}$ | $x = a \sec \theta$ | $\sec^2 \theta - 1 = \tan^2 \theta$ |

4. Integrate the following:

(a) $\int \frac{x^2 - 7x + 1}{x^2 + 1} dx = x - \frac{7}{2} \ln|x^2+1| + C$ (b) $\int \frac{x^3}{\sqrt{5+x^2}} dx = \frac{(5+x^2)^{3/2}}{3} - 5\sqrt{5+x^2} + C$

(c) $\int \sqrt{1 + \cos 2x} dx = \sqrt{2} \sin x + C$ (d) $\int \frac{1}{x^2 - x} dx = \ln|x-1| - \ln|x| + C$

(e) $\int \frac{x^3}{x-1} dx = \frac{x^3}{3} + \frac{x^2}{2} + x + \ln|x-1| + C$ (f) $\int \sqrt{5-9x^2} dx = \frac{5}{6} \left(\sin^{-1} \left(\frac{3x}{\sqrt{5}} \right) + \frac{3x\sqrt{5-9x^2}}{5} \right) + C$

Bonus:

1. (a) $\int \frac{\sqrt{x}}{x-9} dx = 2\sqrt{x} + 3\ln|\sqrt{x}-3| - 3\ln|\sqrt{x}+3| + C$ (b) $\int \frac{\sin x}{\cos^2 x - 2 \cos x} dx = \frac{1}{2} (\ln|\cos x| - \ln|\cos x - 2|) + C$

2. In approximating the integral $\int_a^b f(x) dx$ with n subintervals, define what Δx is.

$\Delta x = \frac{b-a}{n}$