Name: ANSWERS

Instructions: No calculators! Use your own scrap paper and write your answers in the space provided.

1. Complete the following rules:

(a) $\int \ln x \; dx = \underline{x\ln x - x + C}$ (b) $\int \sec x \; dx = \underline{\ln|\sec x + \tan x| + C}$ (c) $\int \tan x \; dx = \underline{-\ln|\cos x| + C}$ or $\underline{\ln|\sec x| + C}$

(d) $\int \cos^2 x \; dx = \underline{\frac{1}{2}\left(x + \frac{1}{2}\sin 2x\right) + C}$ (e) $\int \sin^2 x \; dx = \underline{\frac{1}{2}\left(x - \frac{1}{2}\sin 2x\right) + C}$ (f) $\int \sec^3 x \; dx = \underline{\frac{1}{2}(\sec x \tan x + \ln|\sec x + \tan x|) + C}$

2. Complete the following table of trig substitutions:

Expression	Substitution	Identity
$\sqrt{a^2 - x^2}$	$x = a\sin\theta$ or $x = a\cos\theta$	$1 - \sin^2\theta = \cos^2\theta$ or $1 - \cos^2\theta = \sin^2\theta$
$\sqrt{a^2 + x^2}$	$x = a\tan\theta$	$1 + \tan^2\theta = \sec^2\theta$
$\sqrt{x^2 - a^2}$	$x = a\sec\theta$	$\sec^2\theta - 1 = \tan^2\theta$

3. What is the integration by parts formula?: $\underline{\int u \, dv = uv - \int v \, du.}$

4. What mnemonic is used to choose the parts in the above formula? $\underline{LIATE}$

5. Integrate the following:

(a) $\int_1^4 e^{\sqrt{x}} \; dx = \underline{2e^2}$

(b) $\int \dfrac{x^3}{\sqrt{2^2 + x^2}} \; dx = \underline{\dfrac{(4+x^2)^{3/2}}{3} - 4(4+x^2)^{1/2} + C}$

(c) $\int \dfrac{1 - \tan^2\theta}{\sec^2\theta} \; d\theta = \underline{\frac{1}{2}\sin 2\theta + C}$

(d) $\int \dfrac{x^2}{\sqrt{x^2 + 2}} \; dx = \underline{\dfrac{x\sqrt{x^2+2}}{2} - \ln\left|\dfrac{\sqrt{x^2+2}}{\sqrt{2}} + \dfrac{x}{\sqrt{2}}\right| + C}$

(e) $\int t\sin^2 t \; dt = \underline{\dfrac{t^2}{4} - \dfrac{t}{4}\sin 2t - \dfrac{1}{8}\cos 2t + C}$

(f) $\int \arctan\dfrac{1}{x} \; dx = \underline{x\tan^{-1}\dfrac{1}{x} + \dfrac{1}{2}\ln|1 + x^2| + C}$

(g) $\int \sin^3 x \cos^2 x \; dx = \underline{\dfrac{\cos^5 x}{5} - \dfrac{\cos^3 x}{3} + C}$

Bonus:

1. $\int \dfrac{x^2 + 2}{x^2 + 1} dx = \underline{x + \tan^{-1} x + C}$ (b) $\int \dfrac{1}{x^2 + 3x + 2} dx = \underline{\ln|x+1| - \ln|x+2| + C}$

2. Write down the partial fractions decomposition of $\dfrac{3}{x^2(x^2+1)^2(x^2-4)}$. You need not find the arbitrary constants. $\underline{\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{Cx+D}{x^2+1} + \dfrac{Ex+F}{(x^2+1)^2} + \dfrac{G}{x-2} + \dfrac{H}{x+2}}$