MATH 202 Quiz 5 – Version B

September 29, 2015

Name: ANSWERS

Instructions: No calculators! Use your own scrap paper and write your answers in the space provided.

1. Complete the following identities:

(a) $\cosh^2 x$ $- \sinh^2 x = 1$ (b) $\cos 2x =$ $2\cos^2 x - 1$ (c) $\sin^2 x$ $+ \cos^2 x = 1$

(d) $\sin 2x =$ $2\sin x \cos x$ (e) $\cos(A+B) =$ $\cos A\cos B - \sin A\sin B$ (f) $\sec x =$ $\frac{1}{\cos x}$

(g) $\csc x =$ $\frac{1}{\sin x}$ (h) $1 +$ $\tan^2 x$ $= \sec^2 x$ (i) $\sin(A+B) =$ $\sin A\cos B + \sin B\cos A$

(j) $\cos^2 x =$ $\frac{1+\cos 2x}{2}$ (half-angle formula) (k) $\cosh x =$ $\frac{e^x + e^{-x}}{2}$

2. Compute:

(a) $\cos\frac{7\pi}{6} =$ $-\frac{\sqrt{3}}{2}$ (b) $\tan\frac{5\pi}{4} =$ 1 (c) $\sin\frac{5\pi}{3} =$ $-\frac{\sqrt{3}}{2}$ (d) $\sec 2\pi =$ 1

3. Complete the rules:

(a) $\frac{d}{dx}\cos^{-1} x =$ $\frac{-1}{\sqrt{1-x^2}}$ (b) $\frac{d}{dx}\tan^{-1} u =$ $\frac{u'}{1+u^2}$ (c) $\frac{d}{dx}\sinh x =$ $\cosh x$

4. Differentiate:

(a) $\frac{d}{dx}e^{\arctan^3 x} =$ $\frac{3\arctan^2 x}{1+x^2} \cdot e^{\arctan^3 x}$ (b) $\frac{d}{dx}\cosh(\arcsin\sqrt{x}) =$ $\sinh(\arcsin\sqrt{x}) \cdot \frac{1}{2\sqrt{x}} \cdot \frac{1}{\sqrt{1-x}}$

5. Integrate:

(a) $\int \frac{4e^x}{1+e^{2x}}\,dx =$ $4\arctan(e^x) + C$ (b) $\int \frac{x^3}{\sqrt{1-x^8}}\,dx =$ $\frac{1}{4}\arcsin(x^4) + C$

6. The half-life of cesium-137 is 30 years. Suppose we have a 250 gram sample. Let $P(t)$ be the mass remaining after t years.

(a) Find a differential equation satisfied by $P(t)$: $P' = -\frac{\ln 2}{30}P$ or $P' = \frac{\ln(1/2)}{30}P$

(b) Find $P(t)$ and simplify: $P(t) =$ $250e^{-\frac{\ln 2}{30}t}$ or $P = 250e^{\frac{\ln(1/2)}{30}t}$ or $P = 250\cdot 2^{-t/30}$ or $P = 250\cdot\left(\frac{1}{2}\right)^{t/30}$

(c) How much mass remains after 80 years? $250e^{-\frac{8}{3}\ln 2}$ or $250e^{\frac{8}{3}\ln(1/2)}$ or $250\cdot 2^{-8/3}$ or $250\cdot\left(\frac{1}{2}\right)^{8/3}$

(d) After how many years will the mass be reduced to 1 gram? $\frac{30\ln 250}{\ln 2}$ years

Bonus:

1. $\int x\cos x\,dx =$ $x\sin x + \cos x + C$

2. $\int \sin^2 x\,dx =$ $\frac{1}{2}\left(x - \frac{1}{2}\sin 2x\right) + C$