Name: __ANSWERS__

Instructions: No calculators! Use your own scrap paper and write your answers in the space provided.

1. Complete the following identities:

   (a) $\cosh^2 x - \underline{\sinh^2 x} = 1$ (b) $\sin 2x = \underline{2\sin x \cos x}$ (c) $\underline{\cos^2 x} + \sin^2 x = 1$

   (d) $\cos 2x = \underline{2\cos^2 x - 1}$ $\left[\begin{array}{l}\cos^2 x - \sin^2 x, \text{ OR} \\ 1 - 2\sin^2 x, \text{ OR}\end{array}\right]$ (e) $\sin(A+B) = \underline{\sin A \cos B + \sin B \cos A}$ (f) $\csc x = \underline{\dfrac{1}{\sin x}}$

   (g) $\sec x = \underline{\dfrac{1}{\cos x}}$ (h) $1 + \underline{\tan^2 x} = \sec^2 x$ (i) $\cos(A+B) = \underline{\cos A \cos B - \sin A \sin B}$

   (j) $\sin^2 x = \underline{\dfrac{1 - \cos 2x}{2}}$ (half-angle formula) (k) $\sinh x = \underline{\dfrac{e^x - e^{-x}}{2}}$

2. Compute:
   (a) $\sin\dfrac{7\pi}{6} = \underline{-\tfrac{1}{2}}$ (b) $\tan\dfrac{3\pi}{4} = \underline{-1}$ (c) $\cos\dfrac{5\pi}{3} = \underline{\tfrac{1}{2}}$ (d) $\csc\dfrac{\pi}{2} = \underline{1}$

3. Complete the rules:
   (a) $\dfrac{d}{dx}\sin^{-1}x = \underline{\dfrac{1}{\sqrt{1-x^2}}}$ (b) $\dfrac{d}{dx}\tan^{-1}u = \underline{\dfrac{u'}{1+u^2}}$ (c) $\dfrac{d}{dx}\cosh x = \underline{\sinh x}$

4. Differentiate:
   (a) $\dfrac{d}{dx}e^{\arctan^2 x} = \underline{\dfrac{2\arctan x}{1+x^2}\cdot e^{\arctan^2 x}}$ (b) $\dfrac{d}{dx}\sinh(\arcsin\sqrt{x}) = \underline{\cosh(\arcsin\sqrt{x})\cdot\dfrac{1}{2\sqrt{x}}\cdot\dfrac{1}{\sqrt{1-x}}}$

5. Integrate:

   (a) $\displaystyle\int \dfrac{3e^x}{1+e^{2x}}\,dx = \underline{3\arctan(e^x) + C}$ (b) $\displaystyle\int \dfrac{x^2}{\sqrt{1-x^6}}\,dx = \underline{\tfrac{1}{3}\arcsin(x^3) + C}$

6. The half-life of cesium-137 is 30 years. Suppose we have a 150 gram sample. Let $P(t)$ be the mass remaining after t years.

   (a) Find a differential equation satisfied by $P(t)$: $\underline{P' = -\dfrac{\ln 2}{30}P}$ or $P' = \dfrac{\ln\left(\tfrac{1}{2}\right)}{30}P$

   (b) Find $P(t)$ and simplify: $P(t) = \underline{150\,e^{-\frac{\ln 2}{30}t}}$ or $P = 150\,e^{\frac{\ln(1/2)}{30}t}$ or $P = 150\cdot 2^{-t/30}$ or $P = 150\cdot\left(\tfrac{1}{2}\right)^{t/30}$

   (c) How much mass remains after 80 years? $\underline{150e^{-\frac{8}{3}\ln 2}}$ or $150e^{\frac{8}{3}\ln(1/2)}$ or $150\cdot 2^{-8/3}$ or $150\cdot\left(\tfrac{1}{2}\right)^{8/3}$

   (d) After how many years will the mass be reduced to 1 gram? $\underline{\dfrac{30\ln 150}{\ln 2}}$ years

Bonus:

1. $\displaystyle\int x\sin x\,dx = \underline{-x\cos x + \sin x + C}$

2. $\displaystyle\int \cos^2 x\,dx = \underline{\tfrac{1}{2}\left(x + \tfrac{1}{2}\sin 2x\right) + C}$