

MATH 202 Quiz 4 – Version B

September 17, 2015

Name: ANSWERS

Instructions: No calculators! Use your own scrap paper and write your answers in the space provided.

1. Complete the following rules:

(a)  $(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$  (b)  $\frac{d}{dx} \log_a u = \frac{u'}{u \ln a}$  (c)  $\int a^x dx = \frac{a^x}{\ln a} + C$

(d)  $\frac{d}{dx} e^u = u' e^u$  (e)  $\frac{d}{dx} \ln u = \frac{u'}{u}$  (f)  $\frac{d}{dx} a^x = a^x \ln a$

(g)  $\log_a a^x = x$  (h)  $\log_a \left(\frac{A}{B}\right) = \log_a A - \log_a B$  (i)  $\frac{d}{dx} a^u = u' a^u \ln a$

(j)  $\log_a b = c \Leftrightarrow a^c = b$  (k)  $\log_a (AB) = \log_a A + \log_a B$  (l)  $a^{\log_a x} = x$

2. Differentiate:

(a)  $\frac{d}{dx} (\cos(3^{x^4})) = -\sin(3^{x^4}) \cdot 4x^3 \cdot 3^{x^4} \ln 3$  (b)  $\frac{d}{dx} (\ln 3^{\sin x} + 5^{\cos x}) = \cos x \cdot \ln 3 - \sin x \cdot 5^{\cos x} \cdot \ln 5$

(c)  $\frac{d}{dx} (\log_5 x)^x = \left(\frac{\ln x + 1}{\ln 5}\right) (\log_5 x)^x = \left(\log_5 x + \frac{1}{\ln 5}\right) (\log_5 x)^x$ , etc.

3. Integrate:

(a)  $\int \frac{\log_4(x-3)}{x-3} dx = \frac{\ln 4}{2} (\log_4(x-3))^2 + C$  (b)  $\int_2^e 4^x dx = \frac{4^e - 16}{\ln 4}$

(c)  $\int \frac{\pi^x}{1 + \pi^x} dx = \frac{1}{\ln \pi} \ln |1 + \pi^x| + C$

Bonus:

1. A population, with an initial size of  $P_0$ , grows at a rate proportional to its current size,  $P$ . Assuming its relative growth rate is  $r$ , write down equations for:

(i) The differential equation describing this growth:  $P' = rP$

(ii) The formula for  $P(t)$ , the current size of the population at time  $t$ :  $P(t) = P_0 e^{rt}$

2.  $e = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$  (limit)  $\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$