

MATH 202 Quiz 3 – Version B

September 8, 2015

Name: ANSWERS

Instructions: No calculators! Use your own scrap paper and write your answers in the space provided.

1. Complete the following rules:

(a)  $(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$  (b)  $f(f^{-1}(x)) = x$  (c)  $f^{-1}(f(x)) = x$

(d)  $\frac{d}{dx} e^u = u' e^u$  (e)  $\frac{d}{dx} \ln u = \frac{u'}{u}$  (f)  $\log_a(x^n) = n \log_a x$

(g)  $\log_a a^x = x$  (h)  $\log_a \left(\frac{A}{B}\right) = \log_a A - \log_a B$  (i)  $\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$

(j)  $\log_a b = c \Leftrightarrow a^c = b$  (k)  $\log_a(AB) = \log_a A + \log_a B$  (l)  $a^{\log_a x} = x$

2. Find  $(f^{-1})'(a)$  for the following:

(a)  $f(x) = \sqrt{x^3 + x^2 + x + 1}, a = 2, (f^{-1})'(a) = \frac{2}{3}$

(b)  $f(x) = \cos x, 0 \leq x \leq \pi, a = \frac{\sqrt{2}}, (f^{-1})'(a) = -\sqrt{2}$

3. Find the inverse function of the following:

(a)  $f(x) = \frac{3x-2}{5x+2} \Rightarrow f^{-1}(x) = \frac{2x+2}{3-5x}$  (b)  $f(x) = \sqrt{2x-1} \Rightarrow f^{-1}(x) = \frac{x^2+1}{2}$

4. Differentiate:

(a)  $\frac{d}{dx} (\sin x + (x^3 + 1)^x) = \cos x + (x^3 + 1)^x (\ln(x^3 + 1) + \frac{3x^2}{x^3 + 1})$  (b)  $\frac{d}{dx} \ln \left( \frac{\sqrt{x} \cos x}{(x^5 + 1)^3 \pi^e} \right) = \frac{1}{2x} - \tan x - \frac{15x^4}{x^5 + 1}$

(c)  $\frac{d}{dx} \ln e^{x^2 - 7x + \pi} = 2x - 7$

5. Integrate:

(a)  $\int \frac{(\ln x)^3}{5x} dx = \frac{(\ln x)^4}{20} + C$  (b)  $\int \frac{e^{\frac{1}{x}}}{x^2} dx = -e^{\frac{1}{x}} + C$

(c)  $\int_0^{\pi/4} \frac{\cos x}{1 + 2 \sin x} dx = \frac{1}{2} \ln |1 + \sqrt{2}|$

Bonus:

1.  $\frac{d}{dx} (x^2 e^{2x + \cos x}) = e^{2x + \cos x} (2x + 2x^2 - x^2 \sin x) = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$  (limit)  $\frac{d}{dx} \log_a x = \frac{1}{x \ln a}$

2. Given  $f(\pi^2) = 9$  and  $f'(\pi^2) = 2/7$ , find  $(f^{-1})'(9) = \frac{7}{2}$

3. Suppose  $g(x) = \frac{1}{f^{-1}(x)}, f(3) = 2$  and  $f'(3) = 1/9$ , find  $g'(2) = -1$