

MATH 202 Quiz 3 – Version A

September 8, 2015

Name: ANSWERS

Instructions: No calculators! Use your own scrap paper and write your answers in the space provided.

1. Complete the following rules:

(a) $\frac{d}{dx} e^u = u' e^u$ (b) $\frac{d}{dx} \ln u = \frac{u'}{u}$ (c) $\log_a(x^n) = n \log_a x$

(d) $(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$ (e) $f(f^{-1}(x)) = x$ (f) $f^{-1}(f(x)) = x$

(g) $\log_a b = c \Leftrightarrow a^c = b$ (h) $\log_a(AB) = \log_a A + \log_a B$ (i) $a^{\log_a x} = x$

(j) $\log_a a^x = x$ (k) $\log_a \left(\frac{A}{B}\right) = \log_a A - \log_a B$ (l) $\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$

2. Find the inverse function of the following:

(a) $f(x) = \frac{1+3x}{5-2x} \Rightarrow f^{-1}(x) = \frac{5x-1}{2x+3}$ (b) $f(x) = \sqrt{2+5x} \Rightarrow f^{-1}(x) = \frac{x^2-2}{5}$

3. Find $(f^{-1})'(a)$ for the following:

(a) $f(x) = \sqrt{x^5 - x^3 + 4x}, a = 2, (f^{-1})'(a) = \frac{2}{3}$

(b) $f(x) = \sin x, -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}, a = \frac{\sqrt{3}}{2}, (f^{-1})'(a) = 2$

4. Differentiate:

(a) $\frac{d}{dx} (x^2 + (\cos x)^x) = 2x + (\cos x)^x (\ln \cos x - x \tan x)$ (b) $\frac{d}{dx} \ln \left(\frac{e^{\pi(x^3+1)^2}}{\sqrt{x} \sin x} \right) = \frac{6x^2}{x^3+1} - \frac{1}{2x} - \cot x$

(c) $\frac{d}{dx} e^{\ln(x^3-3x+7)} = 3x^2 - 3$

5. Integrate:

(a) $\int \frac{5}{x(\ln x)^3} dx = -\frac{5}{2} (\ln x)^{-2} + C$ (b) $\int \frac{e^{\frac{1}{x}}}{x^2} dx = -e^{\frac{1}{x}} + C$

(c) $\int_{\pi/4}^{\pi/2} \frac{\sin x}{1+3 \cos x} dx = \frac{1}{3} \ln \left| 1 + \frac{3\sqrt{2}}{2} \right|$

Bonus:

1. $\frac{d}{dx} (x e^{7x^2 + \sin x}) = e^{7x^2 + \sin x} (1 + 14x^2 + x \cos x)$ $\frac{d}{dx} \log_a x = \frac{1}{x \ln a}$ $e = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$ (limit)

2. Given $f(\pi) = 7$ and $f'(\pi) = 3/2$, find $(f^{-1})'(7) = \frac{2}{3}$

3. Suppose $g(x) = \frac{1}{f^{-1}(x)}$, $f(3) = 2$ and $f'(3) = 1/9$, find $g'(2) = -1$