Math 201 Test 2A
October 28, 2014

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Note that both sides of each page may have printed material.

Instructions:

1. Read the instructions.

2. Don't panic! I repeat, do NOT panic!

3. Complete all problems. In this exam, each non-bonus problem is worth 20 points. The weight of the bonus problems are indicated.

4. Show ALL your work to receive full credit. You will get 0 credit for simply writing down the answers (unless it's a case of fill in the blank or state a definition, etc.)

5. Write neatly so that I am able to follow your sequence of steps and box your answers.

6. Read through the exam and complete the problems that are easy (for you) first!

7. No scrap paper, calculators, notes or other outside aids allowed—including divine intervention, telepathy, knowledge osmosis, the smart kid that may be sitting beside you or that friend you might be thinking of texting. In fact, **cell phones should be out of sight!**

8. Use the correct notation and write what you mean! x^2 and $x2$ are not the same thing, for example, and I will grade accordingly.

9. Other than that, have fun and good luck!

Remember: this is the test this class deserves, and also the one it needs right now. *DK theme music*

1. (a) Let $f(x) = 2x - \frac{3}{x}$. Use the limit definition of the derivative to find $f'(x)$. **No credit will be given for any other method!** (15 points)

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{2(x+h) - \frac{3}{x+h} - \left(2x - \frac{3}{x}\right)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{2x} + 2h - \frac{3}{x+h} - \cancel{2x} + \frac{3}{x}}{h}$$

$$= \lim_{h\to 0} \frac{2\cancel{h}}{\cancel{h}} + \lim_{h\to 0} \frac{-\frac{3}{x+h} + \frac{3}{x}}{h} \cdot \frac{x(x+h)}{x(x+h)}$$

$$= 2 + \lim_{h\to 0} \frac{\cancel{-3x} + \cancel{3x} + 3\cancel{h}}{\cancel{h}x(x+h)}$$

$$= 2 + \lim_{h\to 0} \frac{3}{x(x+h)}$$

$$= \boxed{2 + \frac{3}{x^2}}$$

(b) Using your answer to part (a), compute the equation of the tangent line to $f(x)$ at the point where $x = 2$. **Write your line in $y = mx + b$ form.** (5 points)

When $x=2$, $y = 2(2) - \frac{3}{2} = \frac{5}{2}$; also $m = 2 + \frac{3}{2^2} = \frac{11}{4}$

$\Rightarrow$ tangent line is: $y - \frac{5}{2} = \frac{11}{4}(x-2)$

$$\Rightarrow y = \frac{11}{4}x - \frac{11}{2} + \frac{5}{2}$$

$$\Rightarrow \boxed{y = \frac{11}{4}x - 3}$$

2. Find $\frac{dy}{dx} = y'$ for the following. Simplify your answers. (5 points each)

(a) $y = x^2(1-\sin x)^{\sqrt{5}}$

$$\Rightarrow y' = 2x(1-\sin x)^{\sqrt{5}} + x^2 \cdot \sqrt{5}(1-\sin x)^{\sqrt{5}-1} \cdot (-\cos x)$$

$$= \boxed{x(1-\sin x)^{\sqrt{5}-1}\left[2(1-\sin x) - \sqrt{5}\,x\cos x\right]}$$

OR

$$= \boxed{x(1-\sin x)^{\sqrt{5}-1}(2 - 2\sin x - \sqrt{5}\,x\cos x).}$$

(b) $y = 2\sqrt{x} + \frac{5}{\sqrt[3]{x}} - \pi^2 = 2x^{1/2} + 5x^{-1/3} - \pi^2$

$$\Rightarrow y' = \boxed{x^{-1/2} - \frac{5}{3}x^{-4/3}}$$

(c) $y = \frac{4\cos^2 x + x}{\sin^2 x - x}$

$$\Rightarrow y' = \frac{(\sin^2 x - x)(-8\cos x\sin x + 1) - (4\cos^2 x + x)(2\sin x\cos x - 1)}{(\sin^2 x - x)^2}$$

$$= \frac{-8\sin^3 x\cos x + \sin^2 x + 8x\cos x\sin x \cancel{- x} - 8\cos^3 x\sin x + 4\cos^2 x - 2x\sin x\cos x \cancel{+ x}}{(\sin^2 x - x)^2}$$

$$= \boxed{\frac{-8\sin^3 x\cos x + \sin^2 x + 6x\cos x\sin x - 8\cos^3 x\sin x + 4\cos^2 x}{(\sin^2 x - x)^2}}$$

We could also apply $\sin 2x = 2\sin x\cos x$

(d) $x\tan y^2 + xy + \pi = x^3 + 7^2$

$$\Rightarrow \tan y^2 + x \cdot 2y\sec^2 y^2 \frac{dy}{dx} + y + x\frac{dy}{dx} = 3x^2$$

$$\Rightarrow \frac{dy}{dx}(2xy\sec^2 y^2 + x) = 3x^2 - \tan y^2 - y$$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{3x^2 - \tan y^2 - y}{2xy\sec^2 y^2 + x}}$$

3. (5 points each part) A bunch of angry calculus students (allegedly) throw Jhevon off a cliff. Jhevon's position above the ground at time t seconds is given by $s(t) = -16t^2 - 64t + 512$.

(a) Find functions that describe Jhevon's velocity and acceleration at time t.

$s'(t) = \boxed{v(t) = -32t - 64} \rightarrow$ velocity

$s''(t) = \boxed{a(t) = -32} \rightarrow$ acceleration.

(b) What is the highest height that Jhevon attains?

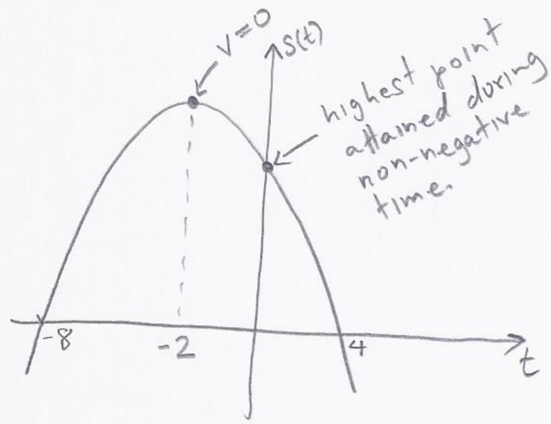
Max height : check $v(t) = 0$

$\Rightarrow -32t - 64 = 0$

$\Rightarrow t = -2 \rightarrow$ impossible

$\Rightarrow \boxed{t = \underset{sec}{0} \text{ for max height}}$

why? $\longrightarrow$



(c) When will Jhevon hit the ground and the nightmare end for his students?

Jhevon hits ground when $s(t) = 0$

$\Rightarrow -16t^2 - 64t + 512 = 0$

$\Rightarrow -16(t^2 + 4t - 32) = 0$

$\Rightarrow -16(t+8)(t-4) = 0$

$\Rightarrow \cancel{t = -8}$ or $\boxed{t = 4 \text{ sec}} \rightarrow$ after this long, Jhevon will hit the ground.

reject!

(d) With what velocity will Jhevon hit the ground? This number shall be commemorated with fond memories.

The required velocity $= v(4)$

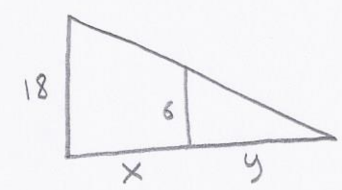
$= -32(4) - 64$

$= \boxed{-192 \text{ ft/sec}}$

As mentioned in class, the instant we care about is when the man is 10 feet from the post. In particular, this is needed for parts (c) and (d). It's better to do (d) first.

4. (5 points each part) A 6-foot tall man walks away from a light post that is 18 feet tall with the light at the very top of it. If the length of the man's shadow is increasing at a rate of 4 feet/second:

(a) How fast is the man walking?



Know: $\frac{dy}{dt} = 4$

Want: $\frac{dx}{dt}$

$$\Rightarrow \frac{x+y}{18} = \frac{y}{6}$$
$$\Rightarrow 6x + 6y = 18y$$
$$\Rightarrow 6x = 12y$$
$$\Rightarrow x = 2y \ *$$
$$\Rightarrow \frac{dx}{dt} = 2\frac{dy}{dt}$$

$$\frac{dx}{dt} = 2(4) = \boxed{8 \text{ ft/sec}}$$

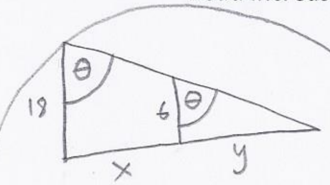
* Or note from the first line that $x + y = 3y$ $\Rightarrow x = 2y$.

(b) How fast is the tip of the man's shadow moving?

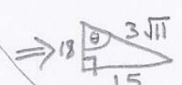
The tip is moving at a rate of $\frac{dx}{dt} + \frac{dy}{dt}$

$$= (4 + 8) \text{ ft/sec}$$
$$= \boxed{12 \text{ ft/sec}}$$

(c) How fast is the angle between the light post and the beam of light that hits the top of the man's head increasing?



Since $x = 2y$ when $x = 10$, $y = 5$



Now $\tan\theta = \frac{y}{6}$

$$\Rightarrow \sec^2\theta \frac{d\theta}{dt} = \frac{1}{6}\frac{dy}{dt}$$
$$\Rightarrow \frac{d\theta}{dt} = \frac{\frac{1}{6}\frac{dy}{dt}}{\sec^2\theta}$$
$$= \frac{\frac{1}{6}\cdot 4}{\left(3\sqrt{11}/18\right)^2}$$

$$\frac{d\theta}{dt} = \frac{18\cdot 18}{99}\cdot\frac{2}{3}$$
$$= \boxed{\frac{24}{11} \text{ rad/sec}}$$

(d) How long is the man's shadow when he is 10 feet away from the light post?

Again, by the equation $x = 2y$ we get $y = 5$ when $x = 10$, so the length of his shadow is 5 feet at that instant.

5. (a) Use linear approximation to approximate $(27.1)^{\frac{1}{3}}$. (8 points)

Take $f(x) = x^{1/3}$ and use $f(x) \approx f(a) + f'(a)(x-a)$ with $x = 27.1$ and $a = 27$.

$$f'(x) = \frac{1}{3}x^{-2/3}$$

$$\Rightarrow f(a) = f(27) = 27^{1/3} = 3$$

$$f'(a) = f'(27) = \frac{1}{3}(27)^{-2/3} = \frac{1}{27}$$

$$\Rightarrow (27.1)^{1/3} \approx 3 + \frac{1}{27}(27.1 - 27)$$

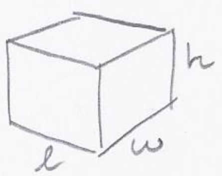
$$= \boxed{3 + \frac{1}{270}} \text{ or}$$

$$= \boxed{\frac{811}{270}}$$

(b) Ashley forgot her glasses, and so when she measures a rectangular box, she could be off by as much as 0.1 cm for each dimension.

(i) Calculate the maximum error in measuring the volume of the box. (8 points)

$V = lwh$ with $dl = dw = dh = 0.1$ cm



$$\Rightarrow dV = dl \cdot wh + dw \cdot lh + dh \cdot lw$$

$$= \boxed{0.1(wh + lh + lw) \text{ cm}^3}$$

(ii) What is the relative error in measuring the volume? (2 points)

$$\text{rel. error} = \boxed{\frac{0.1(wh + lh + lw)}{lwh}}$$

(iii) What is the percentage error in measuring the volume? (2 points)

$$\text{\%-age error} = \frac{0.1(wh + lh + lw)}{lwh} \times 100\% = \boxed{\frac{10(wh + lh + lw)}{lwh}\%}$$

Bonus Problems: (You must complete all problems in the actual test to be eligible).

1. (10 points) The concentration of a drug in a patient's bloodstream t hours after it is taken is given by

$$C(t) = \frac{0.016t}{(t+2)^2}\, mg/cm^3.$$

Find the maximum concentration of the drug and the time at which it occurs.

$$C'(t) = \frac{(t+2)^2 \cdot (0.016) - 0.016t \cdot 2(t+2)}{(t+2)^2}$$

For crit. pt., $C' = 0$ or undefined.

C' is undefined for $t = -2$. Reject this case, t = time.

$$C' = 0 \Rightarrow (t+2)^2 (0.016) - 0.016t \cdot 2(t+2) = 0$$

$$\Rightarrow 0.016(t+2)(t+2-2t) = 0$$

$$\Rightarrow \cancel{t=-2} \quad \text{or} \quad \boxed{t=2} \rightarrow \text{when the max occurs.}$$

reject

The max concentration is $C(2) = \dfrac{0.016(2)}{16}$

$$= \frac{\cancel{16} \cdot 2}{1000 \cdot \cancel{16}}$$

$$= \boxed{\frac{1}{500}\ mg/cm^3}$$

$\hookrightarrow$ max. conc.