Math 212

SYLLABUS AND HOMEWORK

revised 8/2025

Textbook:

Calculus: Early Transcendentals, 9th edition (Cengage)

CONTENTS OF THIS DOCUMENT

PART 1: Two-page table which, in the second column from the left, lists all the sections in the text which are in the **Syllabus**. This list plus the following supplements to three sections from this list form the syllabus for the course:

Notes1a: The Limit Comparison Test for integrals is not in Section 7.8.

Notes5a: Shifts, Reflections and Symmetry, the General Definition of Cones, and Intercepts are not in Section 12.1.

Notes5b: A discussion of why the standard listing of the Quadric Surfaces are the only non-degenerate Quadric Surfaces is not in Section 12.6. The discussion of the Conic Sections in Notes 5b is used for the classification of Quadric Surfaces, but is not all in Appendix C (or Section 10.5).

The column on the right lists suggested **Homework** problems from the textbook for each section. Also, ask your instructor about online homework problems.

PART 2: Ten-page list of **Supplementary Problems**, which is a list of problems that are not in the textbook, but which the student should be able to solve. These problems are marked with a prefix 's'. For example, the problems for Section 3.8 include 's1-3' which refer to problems listed in Part 2.

PART 3: Eleven-page list of **Answers** to all (both text and supplementary) problems that do not have answers in the text. For some problems, **complete solutions** are included.

Suggested Periods	Section	Topics	Exercises
1	3.8	Exponential Change; recall a^x=e^ (xln a); log_a x=(ln x)/ln a (see pp. 59-60 and A57, A59)	5a, 11, 15, s1-3
1	3.11	Hyperbolic Functions	1, 9, 35, 37, s1-2
.5	5.3	Fundamental Theorem of Calculus	s1-4
1.5	5.5	Substitution Theorem	3, 7, 17, 19, 21, 33, 37, 43, 61, 65, 77, s1
1	App. G	Definitiion of e via an Integral	s1-3
1	p. 485	Table of Integrals to Memorize, also ∫sec x and ∫csc x	s1 10
1	9.3	Separable Differential Equations	13, 15, 17, 21, s1
2	7.1	Integration by Parts	19, 21, 29, 33, 34, 37, 43, 37, 43 (Hint: u=√x, u^2=x, 2u du=dx), 9.3/19
1.5	7.2	Trigonometric Integrals	3, 5, 9, 11, 25, 29, 47. s1
2	7.3	Trigonometric Substitution	9, 11, 13, 17, 19, (Hint: tan^2 x=sec^2 x - 1)21 (Hint: 0<= x<=a if 0<=theta<=pi/2), 27
2.5	7.4	Rational Functions	7, 8, 18, 23, 41, 43 (Hint: same as 7.1/43), s1-2
.5	7.5	Guidelines for Integration	5(a)-(b), 7 (Hint: x^3 e^(x^2)= x x^2 e^(x^2), 9, 11, 13, 23, 33, 43 (Hint: symmetry), 45
.5	7.7	Numerical Integration (omit error bounds)	s1-3
2.5	7.8	Improper Integrals	7, 9, 13-21 odd, 27, 37, 57
1	11.1	Sequences	7, 9, 11, 15, 17, 20, 27, 29, 31, 37, 39, 43-49 odd, 53
2	11.2	Series, Geometric Series, Test for Divergence, Telescoping Series	15, 17, 19, 27, 29, 37, 47, 49, s1
2.5	11.3	Integral test	5, 7, 9, 17, 25, 41, s1
2.5	11.4	Comparison Tests	7, 9, 11, 15-31 odd
2	11.5	Alternating Series, Absolute Convergence	5, 7, 11, 13, 19, 23-31 odd, 41, 42, s1
2	11.6	Ratio and Root Test	3-15 odd, 19, 21-27 31, 39, s1
.5	11.7	Guidelines for Testing Series	9-23 odd. 27, 29, 31, 41 (Hint: divide top and bottom 4^n or 5^n)

Suggested Periods	Section	Topics	Exercises
1	11.8	Power Series, Interval of Convergence	3-9 odd, 15, 17, 21, 23, 27, 33 (Hint: factor out 5), s1-2
2.5	11.9	Representing Functions as Power Series	3-9 odd, 23 (find series only), 31 (change limit .3 to .1)
3.5	11.10	Taylor Series, Applications of Power Series	5, 41, 43, 56, 64, s1
2.5	10.3	Polar Coordinates	1-11 odd, 15, 21, 23, only graph polar curves: 33, 35 and 43; s1-3
1	10.4	Polar Integration	3, 7, 9, 11, s1-3
1	App. C	Conic Sections from Second Degree Equations (see notes on 212 webpage)	5, 25, 31, s1-4
2.5	12.1	3-Dimensional Coordinate Systems	5, 7, 17, 19, 27, 31, 39, s1
4	12.6	Quadric Surfaces	13-19 odd, 23-30 all, 33, 37, 39, s1-2

Total: 49 hours

SUPPLEMENTARY HOMEWORK PROBLEMS

3.8/s1. In a certain region, the population, $P(t)$, in thousands of people, t years after census there began, is approximated using an exponential growth model. The initial census showed a population of $P_0 = 90$, and the population two years later was $P(2) = 120$.

(a) Find a formula for $P(t)$.

(b) Find the population after 4 years. Simplify the answer, which is an integer.

(c) Find the population after 5 years. (The answer is not an integer.)

(d) How long does it take for the population to double?

(e) How long (i.e., how many years) will it take for the population to reach a million people?

3.8/s2. Two years after opening a bank account in which interest is at 4% compounded continuously, the balance is \$542.

(a) What was the opening balance of the account?

(b) Find the balance, $A(t)$, as function of time t in years after opening.

3.8/s3. A culture of bacteria which is placed in a dish has grown to 6 grams after two hours and 24 grams after six hours.

(a) Find the initial amount of bacteria, expressed an integer.

(b) Find the function, $A(t)$, of the number of grams in the dish t hours after being placed in the dish.

(c) Find how many hours after being placed in the dish there will be 20 grams of bacteria.

3.11/s1. Find $\cosh x$ when $\sinh x = 2$.

3.11/s2. Find $\dfrac{d}{dx}[x^2 \ln(\sinh(5x))]$.

5.3/s1. (a) Differentiate $\dfrac{x^2}{x^5 + x + 1}$.

(b) Use part (a) to evaluate $\displaystyle\int \frac{-3x^6 + x^2 + 2x}{(x^5 + x + 1)^2}\,dx$.

5.3/s2. (a) Differentiate $\sin(\sqrt[3]{x^4})$

(b) Use part (a) to evaluate: $\displaystyle\int \sqrt[3]{x}\cos(\sqrt[3]{x^4})\,dx$.

5.3/s3. (a) Differentiate $(x^3 + x + 1)e^{2x}$.

(b) Use part (a) to evaluate $\displaystyle\int (2x^3 + 3x^2 + 2x + 3)e^{2x}\,dx$.

5.3/s4. (a) Differentiate $\dfrac{\sinh x}{x^2 + 1}$.

(b) Use part (a) to evaluate $\displaystyle\int \frac{(x^2 + 1)\cosh x - 2x\sinh x}{(x^2 + 1)^2}\,dx$.

5.5/s1. Evaluate: $\displaystyle\int x^2 \sinh(x^3)\,dx$

App. G/s1. Evaluate: $\displaystyle\int_1^2 2^{3x}\,dx$

App. G/s2. Evaluate: $\displaystyle\int \frac{y}{y^2 - 25}\,dx$

App. G/s3. Evaluate: $\displaystyle\int_e^{e^4} \frac{1}{x\log_4 x}\,dx$

p. 485/s1. Evaluate: $\displaystyle\int_e^{e^4} \frac{1}{x(\ln x)^2}\,dx$

p. 485/s2. Evaluate: $\displaystyle\int e^{2x}\sec^2 e^{2x}\,dx$

p. 485/s3. Evaluate: $\displaystyle\int \tan(3x + 1)\,dx$

p. 485/s4. Evaluate: $\displaystyle\int (\sec x\tan x)e^{\sec x}\,dx$

p. 485/s5. Evaluate: $\displaystyle\int (2 + e^x)^5 e^x\,dx$

p. 485/s6. Evaluate: $\displaystyle\int (\sec^2 x)[\tan(\tan x)]\,dx$

p. 485/s7. Evaluate: $\displaystyle\int \sqrt{x}\sec(x\sqrt{x})\,dx$

p. 485/s8. Evaluate: $\displaystyle\int \frac{e^x}{1+e^{2x}}\,dx$

p. 485/s9. Evaluate: $\displaystyle\int_0^{\sqrt{\pi/4}} 4x\tan(x^2)\,dx$

p. 485/s10. Evaluate: $\displaystyle\int \frac{\sec(\ln x)}{x}\,dx$

9.3/s1. Find a function $y(x)$ such that $y' = xy^2e^{x^2-1}$ and $y(1) = -1$.

7.2/s1. Evaluate: $\int \cot^2 x\,dx$

7.4/s1. Evaluate: $\displaystyle\int \frac{3x^2+3x-2}{x^3-x^2}\,dx$

7.4/s2. Evaluate: $\displaystyle\int_3^4 \frac{3x-4}{x^3-3x^2+2x}\,dx$

7.7/s1. Approximate $\displaystyle\int_0^4 \frac{12x}{x+1}\,dx$ using $n = 4$ subintervals and (a) the Trapezoidal Rule and (b) Simpson's Parabolic Rule.

7.7/s2. Approximate $\displaystyle\int_0^3 x^3\,dx$ using $n = 6$ subintervals and (a) the Trapezoidal Rule and (b) Simpson's Parabolic Rule.

7.7/s3. Approximate $\displaystyle\int_1^9 x^4\,dx$ using $n = 4$ subintervals and (a) the Trapezoidal Rule and (b) Simpson's Parabolic Rule.

For Chapter 10, see after Chapter 11.

11.2/s1. Are the series below convergent or divergent? Find the sum of any that are convergent.

(a) $\displaystyle\sum_{n=4}^{\infty} \frac{1}{n}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{4n}$ (c) $\displaystyle\sum_{n=2}^{\infty} \left(\frac{1}{n^2} - \frac{1}{(n+1)^2} - \frac{(-1)^n}{3^n}\right)$

11.3/s1. (a) Find an upper bound for the error in the approximation $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3} \approx \sum_{n=1}^{4} \frac{1}{n^3}$.

(b) Find a value of N such that the error in the approximation $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3} \approx \sum_{n=1}^{N} \frac{1}{n^3}$ is less than 10^{-4}.

11.5/s1. For the series below state whether they are absolutely convergent, conditionally convergent, or divergent. State which test(s) you used, and explain why the test(s) work.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(n^2+n+1)}{n^2+9}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{n^3+9}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(n^2+n+1)}{n^4+9}$ (d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \sin(n^2)}{n\sqrt{n}}$

11.6/s1. For what value(s) of the constant c will the ratio test show sum below is convergent? For what value(s) of c will the ratio test fail?

$$\sum_{n=1}^{\infty} \frac{n!c^n}{n^n}$$

11.8/s1. Find the interval of convergence: $\displaystyle\sum_{n=0}^{\infty} \frac{(x+3)^n}{(n+1)^2 3^n}$

Remember to check the endpoints if applicable.

11.8/s2. Find the interval of convergence: $\displaystyle\sum_{n=0}^{\infty} \frac{(x-4)^n}{\ln(n+2)}$

Remember to check the endpoints if applicable.

11.10/s1. (a) Find the first five terms of the Maclaurin series (i.e., the power series centered at zero) for $f(x) = \dfrac{1}{2x+1}$.

(b) Find the first four nonzero terms for the Maclaurin series for the derivative, $f'(x)$, of $f(x)$.

(c) Use the answer to (b) to approximate $f'(.05)$, with an error not to exceed .01, and verify that your answer has the required accuracy.

10.3/s1. Graph the polar equation $r = 3 - 2\cos\theta$, and label all x- and y-intercepts, if any exist.

10.3/s2. Graph the polar equation $r = 3 + 6\sin\theta$, and label all x- and y-intercepts, if any exist.

10.3/s3. Graph the region in the first quadrant bounded by the x-axis, the line $y = x$ and $r = 2 + \sin\theta$.

10.4/s1. Graph and find the area of the region in the first quadrant which is outside the graph of $r = 2$ and inside the larger loop of $r = 4 + 2\cos\theta$.

10.4/s2. Graph and find the area of the region outside graph of $r = 2\cos\theta$ and inside the graph of $r = 3 + \cos\theta$.

10.4/s3. Graph and find the area of the region to the right of the y-axis which is outside the graph of $y = -2\sin\theta$ and inside the graph of $r = 4$.

10.5/s1. Graph the equation, labelling the coordinates of the center and all vertices: $x^2 + 4y^2 - 6x + 5 = 0$.

10.5/s2. Graph the equation, labelling the coordinates of the center and all vertices: $4x^2 + y^2 + 2y = 0$.

10.5/s3. Graph the equation, labelling the coordinates of the center and all vertices: $2x^2 - y^2 - 12x + 10 = 0$.

10.5/s4. Graph the equation, labelling the coordinates of the center and all vertices: $-4x^2 + y^2 - 8x - 8 = 0$.

12.1/s1. For the equation $x^2 - 2(y-2)^2 - e^{z^2} = 0$,

(a) show whether or not it has symmetry about (i) the x-axis, (ii) the y-axis, (iii) the xy-plane, (iv) the xz-plane;

(b) graph the trace of the equation in the xy-plane, labelling the coordinates of the center and the vertex (vertices).

12.6/s1. Graph and label all vertices, if any exist:

$$x^2 + 4y^2 - z^2 - 6x + 4z + 9 = 0$$

12.6/s2. (a) Graph and label all vertices, if any exist:

$$2x^2 + 27z^2 + 4x - 16 = 0$$

scroll down for answers and selected solutions

ANSWERS AND SELECTED SOLUTIONS TO EVEN NUMBERED AND SUPPLEMENTARY HOMEWORK PROBLEMS

3.8/s1. In a certain region, the population, $P(t)$, in thousands of people, t years after census there began, is approximated using an exponential growth model. The initial census showed a population of $P_0 = 90$, and the population two years later was $P(2) = 120$.

(a) Find a formula for $P(t)$.

(b) Find the population after 4 years. Simplify the answer, which is an integer.

(c) Find the population after 5 years. (The answer is not an integer.)

(d) How long does it take for the population to double?

(e) How long (i.e., how many years) will it take for the population to reach a million people?

Solution

(a) $P(t) = P_0\left(\frac{P(2)}{P(0)}\right)^{t/(2-0)} = 90\left(\frac{120}{90}\right)^{t/2} = 90\left(\frac{4}{3}\right)^{t/2}$ or $P(t) = 90e^{(t/2)\ln(4/3)}$.

(b) $P(4) = 90(4/3)^{4/2} = 160$ or $P(4) = 90e^{[\ln(4/3)]2} = 90e^{\ln(16/9)} = 90(16/9) = 160$.

(c) $P(5) = 90(4/3)^{5/2} = 90(32/(9\sqrt{3}) = 320/\sqrt{3}$.

(d) Let t_d be the number of years it takes the population to double. $2P_0 = P_0(4/3)^{t_d/2}$. Equate the log of both sides: $\ln 2 = (t_d/2)\ln(4/3)$;

$$t_d = 2\ln 2/\ln(4/3) = \ln 4/\ln(4/3).$$

(e) One million is 1000 thousands. $1000 = 90(4/3)^{t/2}$; $100/9 = (4/3)^{t/2}$; $t = 2\ln(100/9)/\ln(4/3)$.

3.8/s2. Two years after opening a bank account in which interest is at 4% compounded continuously, the balance is \$542.

(a) What was the opening balance of the account?

(b) Find the balance, $A(t)$, as function of time t in years after opening.

Solution

(a) $A(t) = A_0e^{.04t}$; $\qquad 542 = A_0e^{.08}$; $\qquad A_0 = 542/e^{.08}$

(b) $A(t) = 542e^{-.08}e^{.04t} = 542e^{.04t-.08}$

3.8/s3. A culture of bacteria which is placed in a dish has grown to 6 grams after two hours and 24 grams after six hours.

(a) Find the initial amount of bacteria, expressed an integer.

(b) Find the function, $A(t)$, of the number of grams in the dish t hours after being placed in the dish.

(c) Find how many hours after being placed in the dish there will be 20 grams of bacteria.

Solution

(a) $A(t) = A_0 \left(\frac{24}{6}\right)^{t/(6-2)} = A_0 4^{t/4} = A_0 2^{t/2}$.

$6 = A(2) = A_0 2^{2/2}$; $A_0 = 3$.

(b) $A(t) = 3(2^{t/2})$.

(c) $20 = 3(2^{t/2})$; $\frac{20}{3} = 2^{t/2}$; $\ln \frac{20}{3} = (t/2)\ln 2$; $t = \frac{2\ln(20/3)}{\ln 2}$.

3.11/s1. Find $\cosh x$ when $\sinh x = 2$.

Solution

$\cosh^2 x - 2^2 = 1$ and $x > 0$; $\cosh x = \sqrt{5}$

3.11/s2. $2x \ln(\sinh(5x)) + 5x^2 \coth(5x)$

5.3/s1. (a) $\dfrac{-3x^6 + x^2 + 2x}{(x^5 + x + 1)^2}$

(b) $\dfrac{x^2}{x^5 + x + 1} + C$

5.3/s2. (a) $\frac{4}{3}\sqrt[3]{x}\cos(\sqrt[3]{x^4})$

(b) $\int \sqrt[3]{x}\cos(\sqrt[3]{x^4})\,dx = \frac{3}{4}\int \frac{4}{3}\sqrt[3]{x}\cos(\sqrt[3]{x^4})\,dx = \frac{3}{4}\sin(\sqrt[3]{x^4}) + C$

5.3/s3. (a) $(2x^3 + 3x^2 + 2x + 3)e^{2x}$

(b) $(x^3 + x + 1)e^{2x} + C$.

5.3/s4. (a) $\dfrac{(x^2+1)\cosh x - \sinh x}{(x^2+1)^2}$.

(b) $\dfrac{\sinh x}{x^2+1} + C$.

5.5/s1. $\dfrac{1}{3}\cosh(x^3) + C$

App. G/s1. $\dfrac{56}{3\ln 2}$

App. G/s2. $\frac{1}{2}\ln|y^2 - 25| + C$

App. G/s3. Evaluate: $\displaystyle\int_e^{e^4} \frac{1}{x\log_4 x}\,dx.$

Solution

Let $u = \ln x$; $du = \frac{1}{x}\,dx$; $u(e) = 1$; and $u(e^4) = 4$, so

$$\int_e^{e^4} \frac{1}{x\log_4 x}\,dx = \int_e^{e^4} \frac{\ln 4}{x\ln x}\,dx = \textstyle\int_1^4 \frac{\ln 4}{u}\,du = (\ln 4)(\ln x)|_1^4 = (\ln 4)^2.$$

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p. 485/s1. $\dfrac{3}{4}$

p. 485/s2. $\dfrac{1}{2}\tan(e^{2x}) + C$

p. 485/s3. $\dfrac{1}{3}\ln|\sec(3x+1)| + C$

p. 485/s4. $e^{\sec x} + C$

p. 485/s5. $\dfrac{(2+e^x)^6}{6} + C$

p. 485/s6. $\ln|\sec(\tan x)| + C$

p. 485/s7. $\dfrac{2}{3}\ln|\sec(x\sqrt{x}) + \tan(x\sqrt{x})| + C$

p. 485/s8. $\tan^{-1}(e^x) + C$

p. 485/s9. $2\ln(\sqrt{2}) = \ln 2$

p. 485/s10. $\ln|\tan(\ln x) + \sec(\ln x)| + C$

9.3/s1. Find a function $y(x)$ such that $y' = xy^2e^{x^2-1}$ and $y(1) = -1$.

Solution

Let $u = x^2 - 1$; $du = 2x\,dx$:

$$\int \frac{1}{y^2}\,dy = \int xe^{x^2-1}\,dx$$

$$-\frac{1}{y} = \int e^u\left(\frac{1}{2}du\right) = \frac{1}{2}e^{x^2-1} + C$$

$$y(1) = -1: \quad 1 = \frac{1}{2} + C, \text{ so } C = \frac{1}{2}$$

$$y = -\frac{1}{(1/2)e^{x^2-1} + (1/2)} = \frac{-2}{e^{x^2-1}+1}$$

7.1/34. Evaluate: $\displaystyle\int_0^{2\pi} t^2 \sin 2t\,dt$

Solution

Integrate by parts:

$$= t^2\left(-\frac{1}{2}\cos 2t\right)\Big|_0^{2\pi} - \int_0^{2\pi} 2t\left(-\frac{1}{2}\cos 2t\right)dt$$

$$= -2\pi^2 + \int_0^{2\pi} t\cos 2t\,dt$$

$$= -2\pi^2 + t\left(\frac{1}{2}\sin 2t\right)\Big|_0^{2\pi} - \frac{1}{2}\int_0^{2\pi}\sin 2t\,dt = -2\pi^2.$$

The last integral is zero because it is integrated over a whole number of periods of $\sin 2t$.

7.2/s1. $\int \cot^2 x\,dx = \int \sec^2 x - 1\,dx = \tan x - x + C.$

7.4/8. $3\ln|x| - 2\ln|x-4| + C.$

7.4/18. $3x - 5\ln|x+1| + 2\ln|x+2| + C$

7.4/s1. $\ln|x| - \dfrac{2}{x} + 4\ln|x-1| + C$

7.4/s2. $\ln\left|\dfrac{(x-1)(x-2)}{x^2}\right|_3^4 = \ln\dfrac{3}{8} - \ln\dfrac{2}{9} = \ln\dfrac{27}{16}.$

7.7/s1. (a) $\frac{1}{2}\left[\frac{12\cdot 0}{0+1}+2\frac{12\cdot 1}{1+1}+2\frac{12\cdot 2}{2+1}+2\frac{12\cdot 3}{3+1}+\frac{12\cdot 4}{4+1}\right]$

(b) $\frac{1}{3}\left[\frac{12\cdot 0}{0+1}+4\frac{12\cdot 1}{1+1}+2\frac{12\cdot 2}{2+1}+4\frac{12\cdot 3}{3+1}+\frac{12\cdot 4}{4+1}\right]$

Do not simplify!

7.7/s2.

(a) $\frac{1}{4}\left[0^3+2\left(\frac{1}{2}\right)^3+2(1^3)+2\left(\frac{3}{2}\right)^3+2(2^3)+2\left(\frac{5}{2}\right)^3+(3^3)\right]$

(b) $\frac{1}{6}\left[0^3+4\left(\frac{1}{2}\right)^3+2(1^3)+4\left(\frac{3}{2}\right)^3+2(2^3)+4\left(\frac{5}{2}\right)^3+(3^3)\right]$

Do not simplify!

7.7/s3. (a) $\left[1^4+2(3^4)+2(5^4)+2(7^4)+9^4\right]$

(b) $\frac{2}{3}\left[1^4+4(3^4)+2(5^4)+4(7^4)+9^4\right]$

Do not simplify!

For Chapter 10, see after Chapter 11.

11.2/s1. (a) Div; p-series (harmonic series); $p=1\leq 1$. Convergence is the same for all values of the lower limit.

(b) $\frac{1}{4}\sum\frac{1}{n}$; Div; p-series; $p=1$.

(c) $\sum_2^\infty\left(\frac{1}{n^2}-\frac{1}{(n+1)^2}\right)-\sum_2^\infty\frac{(-1)^n}{3^n}=\frac{1}{2^2}-\frac{1/9}{1-(-1/3)}=\frac{1}{6}$;

$\sum(a_n+b_n)=\sum a_n+\sum b_n$; telescoping series and geometric series.

11.3/s1. (a) $R_4\leq\int_4^\infty\frac{1}{x^3}\,dx=\frac{-1}{2x^2}\Big|_4^\infty=\frac{1}{32}$.

(b) $R_N\leq\int_N^\infty\frac{1}{x^3}\,dx=\frac{-1}{2x^2}\Big|_N^\infty=\frac{1}{2N^2}$.

So $R_N\leq\frac{1}{10^4}$ if $\frac{1}{2N^2}\leq\frac{1}{10^4}$, which will be true if $2N^2\geq 10^4$, $\sqrt{2}N\geq 100$, or $N\geq\frac{100}{\sqrt{2}}\approx 71$.

11.5/s1. For each series, then nth term will be denoted by a_n

(a) Div; Test for Divergence; $\lim |a_n| = \lim \dfrac{n^2+n+1}{n^2+9} = \lim \dfrac{n^2}{n^2} = 1$, because $n, 1 << n^2$.

(b) CC; AST and Limit Comparison Test; the series is clearly alternating and has limit 0. Also, for $f(x) = \dfrac{x^2}{x^3+9}$,

$$f'(x) = \frac{2x(x^3+9) - x^2(3x^2)}{(x^3+9)^2} = -\frac{x(x^2-18)}{(x^3+9)^2},$$

which is negative for $x \geq 5$, so $|a_n|$ is eventually decreasing. Thus, the series is convergent (= AC or CC). The Limit Comparison Test applied to $|a_n|$ and $b_n = \dfrac{1}{n}$, shows $\sum |a_n|$ is divergent and $\sum a_n$ is CC.

(c) AC; Limit Comparison Test; For $b_n = \dfrac{1}{n^2}$, $\lim \dfrac{|a_n|}{b_n} = 1$.

(d) AC; Direct Comparison Test; $|a_n| \leq \dfrac{1}{n^2}$.

11.5/42. $1 + \dfrac{1}{2^6} + \dfrac{1}{3^6} + \dfrac{1}{4^6}$; $|R_4| \leq \dfrac{1}{5^6}$ $\left(5^6 = \dfrac{10^6}{2^6} = \dfrac{10^2}{64}10^4\right)$

11.6/s1. For what value(s) of the constant c will the ratio test show the sum below is convergent? For what value(s) of c will the ratio test fail?

$$\sum_{n=1}^{\infty} \frac{n!c^n}{n^n}$$

Solution

The nth term of the series will be denoted by a_n.

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{(n+1)!c^{n+1}}{(n+1)^{n+1}} \frac{n^n}{n!c^n} = \frac{(n+1)cn^n}{(n+1)^{n+1}}$$
$$= \frac{cn^n}{(n+1)^n} = c\left(\frac{n}{n+1}\right)^n = \frac{c}{\left(\frac{n+1}{n}\right)^n} \longrightarrow \frac{c}{e}.$$

The Ratio Test shows that, if $c < e$, then the series is convergent. If $c = e$, the Ratio Test fails. If $c > e$ the series diverges.

11.8/s1. $[-6, 0]$

11.8/s2. By L'Hôpital's Rule,

$$L = \lim \frac{\ln(n+2)}{\ln(n+3)} = \lim \frac{\ln(x+2)}{\ln(x+3)} = \lim \frac{1/(x+2)}{1/(x+3)} = 1.$$

$R = 1/L = 1$. The interval of convergence is $[3, 5)$.

11.10/s1. (a) Find the first five terms of the Maclaurin series (i.e., the power series centered at zero) for $f(x) = \dfrac{1}{2x+1}$.

(b) Find the first four nonzero terms for the Maclaurin series for the derivative, $f'(x)$, of $f(x)$.

(c) Use the answer to (b) to approximate $f'(.05)$, with an error not to exceed .01, and verify that your answer has the required accuracy.

Solution

(a) Write $f(x)$ in the form $f(x) = \dfrac{1}{1-(-2x)}$ and substitute into the equation $\dfrac{1}{1-x} = \sum_{n=0}^{\infty} x^n$:

$$\begin{aligned} f(x) &= \sum_{n=0}^{\infty} (-2x)^n \\ &= \sum_{n=0}^{\infty} (-1)^n 2^n x^n \\ &= 1 - 2x + 2^2x^2 - 2^3x^3 + 2^4x^4 \pm \ldots \end{aligned}$$

$$\begin{aligned} \text{(b)}\, f'(x) &= \sum_{n=0}^{\infty} (-1)^n 2^n n x^{n-1} \\ &= -2 + 2^2 \cdot 2x - 2^3 \cdot 3x^2 + 2^4 \cdot 4x^3 \pm \ldots \\ &= -2 + 8x - 24x^2 + 64x^3 \pm \ldots \end{aligned}$$

$$\text{(c)}\, f'(\tfrac{1}{20}) = -2 + \frac{8}{20} - \frac{24}{400} + \frac{64}{8000} \pm \ldots$$

$$= -2 + \frac{4}{10} - \frac{6}{100} + \frac{8}{1000} \pm \cdots = \boxed{-2 + .4 - .06} - \frac{8}{1000} \pm \ldots$$

$$\approx -1.66 \qquad \leftarrow \text{Answer}$$

$$|R_4| \le \frac{8}{1000} = .008 < .01$$

11.10/56. $1 - \dfrac{1}{10} + \dfrac{1}{2!10^2} + \dfrac{1}{3!10^3}$; $|R_3| \le \dfrac{1}{4!10^4}$

11.10/64. $\dfrac{1}{5} - \dfrac{1}{3!13} + \dfrac{1}{5!17}$; $|R_2| \leq \dfrac{1}{7!21}$

10.3/s1. Graph the polar equation $r = 3 - 2\cos\theta$, and label all x- and y-intercepts, if any exist.

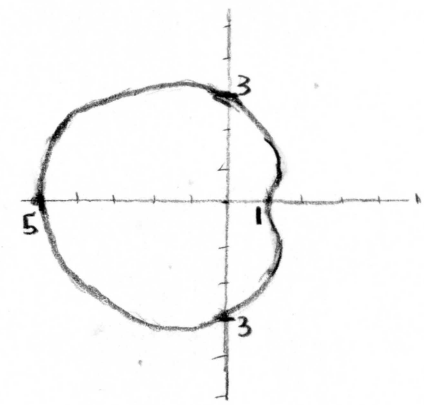
Solution

$r(0) = 1$

$r(\pi/2) = 3$

$r(\pi) = 5$

$r(3\pi/2) = 3$



10.3/s2. Graph the polar equation $r = 3 + 6\sin\theta$, and label all y-intercepts, if any exist.

Solution

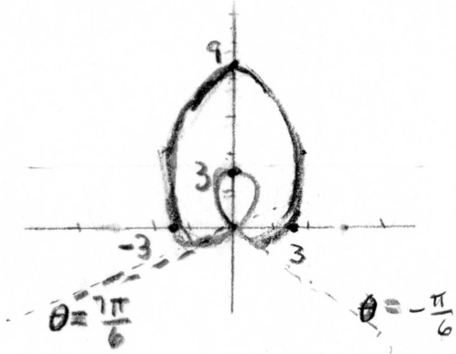
$r(0) = 3$

$r(\pi/2) = 9$

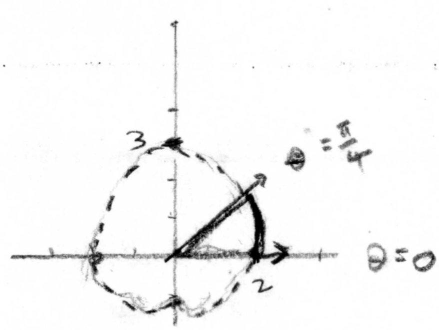
$r(\pi) = 3$

$r(3\pi/2) = -3$

$0 = r(\theta) = 3 + 6\sin\theta$ if $\theta = \dfrac{7\pi}{6}, -\dfrac{\pi}{6}$



10.3/s3. The line $y = x$ has polar equaion $\theta = \dfrac{\pi}{4}$.



10.4/s1. Graph and find the area of the region in the first quadrant which is outside the graph of $r = 2$ and inside the graph of $r = 4 + 2\cos\theta$.

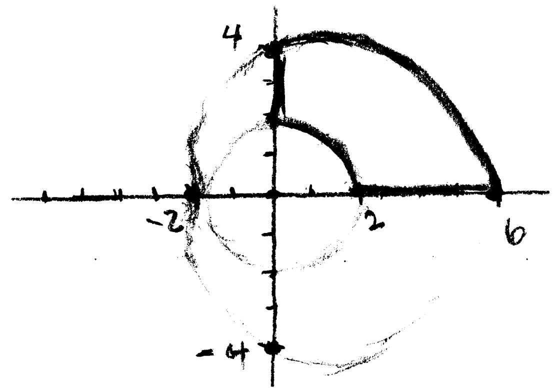
Solution

$r(0) = 6$

$r(\pi/2) = 4$

$r(\pi) = 2$

$r(3\pi/2) = 4$



We need the area, calculated below, in the first quadrant inside the limaçon less $\frac{1}{4}(\pi 2^2)$, the area of the quarter of the circle in the first quadrant.

$$\frac{1}{2}\int_0^{\pi/2} (4 + 2\cos\theta)^2\, d\theta = \frac{1}{2}\int_0^{\pi/2} 2^2(2 + \cos\theta)^2\, d\theta$$

$$= 2\int_0^{\pi/2} 4 + 4\cos\theta + \cos^2\theta\, d\theta$$

$$= 4\pi + 8\sin\theta\Big|_0^{\pi/2} + \int_0^{\pi/2} 1 + \cos 2\theta\, d\theta$$

$$= 4\pi + 8 + \frac{\pi}{2} + \frac{1}{2}\sin 2\theta\Big|_0^{\pi/2} = \frac{9\pi}{2} + 8.$$

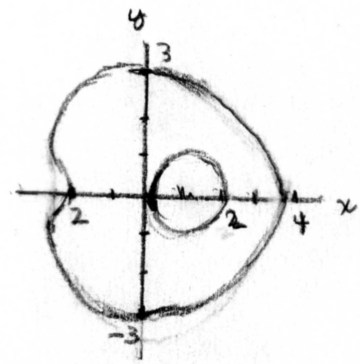
The required area is $\dfrac{7\pi}{2} + 8$

10.4/s2. Graph and find the area of the region outside graph of $r = 2\cos\theta$ and inside the graph of $r = 3 + \cos\theta$.

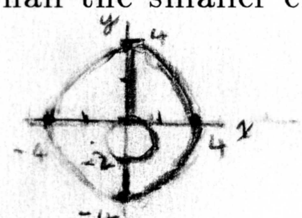
Solution

We calculate the area inside the limacon and subtract the area, π, of the circle. Twice we use the fact that the integral of a $\sin c\theta$ or $\cos c\theta$ over a whole number of periods is 0.

$$\begin{aligned}
&\frac{1}{2}\int_0^{2\pi} (3 + \cos\theta)^2 d\theta - \pi \\
=&\frac{1}{2}\int_0^{2\pi} 9 + 6\cos\theta + \cos^2\theta \, d\theta - \pi \\
=&9\pi + \frac{1}{4}\int_0^{2\pi} 1 + \cos 2\theta \, d\theta - \pi \\
=&9\pi + \frac{\pi}{2} - \pi = \frac{17\pi}{2}.
\end{aligned}$$

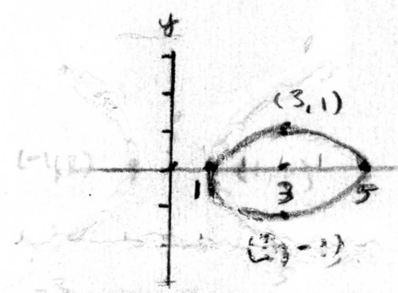


10.4/s3. The area of half the large circle minus half the smaller circle: $\frac{1}{2}(\pi 4^2 - \pi 1^2) = \frac{15\pi}{2}$.



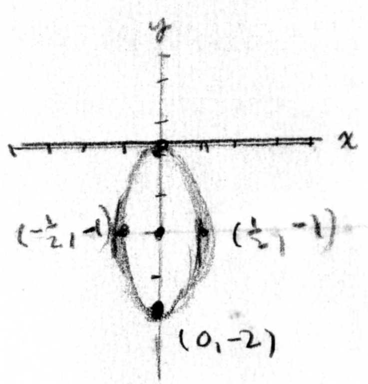
10.5/s1.

$$\frac{(x-3)^2}{4} + y^2 = 1$$



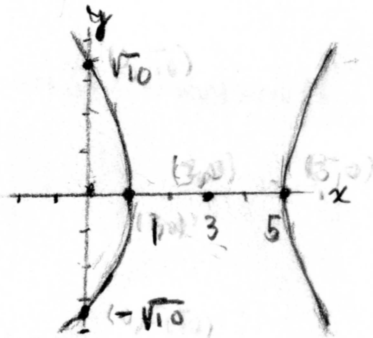
10.5/s2.

$$\frac{x^2}{1/4} + (y+1)^2 = 1$$



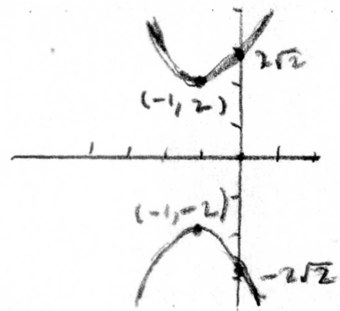
10.5/s3.

$$\frac{(x-3)^2}{4} - \frac{y^2}{8} = 1$$



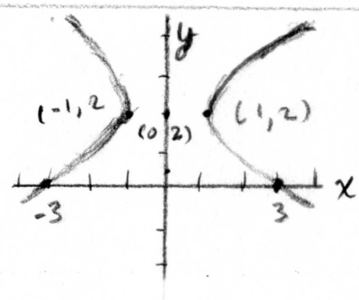
10.5/s4.

$$-(x+1)^2 + \frac{y^2}{4} = 1$$



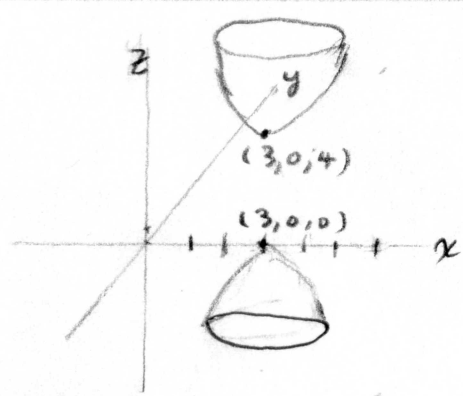
12.1/s1. (a) (i) No;
(ii) Yes; (iii) Yes; (iv) No

(b) $x^2 - 2(y-2)^2 - 1 = 0$,



12.6/s1.

$$-\frac{(x-3)^2}{4} - y^2 + \frac{(z-2)^2}{4} = 1$$



12.6/s2.

$$\frac{(x+1)^2}{9} + \frac{z^2}{2/3} = 1$$