

Name: ANSWERS

Instructions: No calculators! Answer all problems in the space provided! Do your rough work on scrap paper.

1. For the matrix  $A = [a_{ij}] = \begin{pmatrix} 7 & 2 & 3 \\ 5 & 0 & -1 \\ 6 & 7 & \pi \end{pmatrix}$ , what is  $a_{32} =$  7?

2. Let  $A = \begin{pmatrix} 1 & -2 & 2 \\ 3 & 0 & 1 \end{pmatrix}$ ,  $B = \begin{pmatrix} 1 & 0 & 9 \\ -1 & 1 & 5 \\ 3 & 4 & 7 \end{pmatrix}$ ,  $C = \begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}$  and  $D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ . Compute the following, or write "DNE", for "does not exist".

(a)  $B + 2D =$  DNE      (b)  $AB =$   $\begin{pmatrix} 9 & 6 & 13 \\ 6 & 4 & 34 \end{pmatrix}$

(c)  $BA =$  DNE      (d)  $A - 3D =$   $\begin{pmatrix} -2 & -2 & 2 \\ 3 & 0 & -2 \end{pmatrix}$

3. Suppose  $A$  and  $C$  above were multiplied to find  $CA$ . Write the size of the result, or "DNE" if they actually cannot be multiplied:  $2 \times 3$

4. List the square matrices in problem 2.  $B, C$

5. Solve the system  $\begin{matrix} x + 2y - z & = & -2 \\ x & + & z & = & 2 \\ 2x - 4y + z & = & 7 \end{matrix}$  by doing the following:

(a) Write down the augmented matrix for the system:

$$\left( \begin{array}{ccc|c} 1 & 2 & -1 & -2 \\ 1 & 0 & 1 & 2 \\ 2 & -4 & 1 & 7 \end{array} \right)$$

(b) Find the reduced row-echelon form of the augmented matrix:

$$\begin{array}{l} \left( \begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 2 & -2 & -4 \\ 0 & 8 & -3 & -11 \end{array} \right) \begin{array}{l} R_2 \\ R_1 - R_2 \\ 2R_1 - R_3 \end{array} \\ \hline \left( \begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & -5 & -5 \end{array} \right) \begin{array}{l} R_1 \\ R_2/2 \\ 4R_2 - R_3 \end{array} \\ \hline \left( \begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array} \right) \begin{array}{l} R_3/5 + R_1 \\ R_3/-5 + R_2 \\ R_3/-5 \end{array} \end{array}$$

(c) Write down the solution as a column vector:  $\begin{pmatrix} x \\ y \\ z \end{pmatrix} =$   $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

Bonus: (a) If  $A = \begin{pmatrix} 2 & -2 \\ 1 & 0 \end{pmatrix}$ , find  $A^{-1} =$   $\frac{1}{2} \begin{pmatrix} 0 & 2 \\ -1 & 2 \end{pmatrix}$  (b) If  $B = \begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 2 & 1 & 0 \end{pmatrix}$ , find  $\det B =$  2