

Math 392 - Practice Problems for Exam 1

1) Evaluate the integral

$$\int_C (x e^{yz}) ds$$

where C is the straight line segment joining $(0, 0, 0)$ to $(1, 2, 3)$.

2) Find the work done by the force field $\vec{F} = (2y^{3/2}, 3x\sqrt{y})$ in moving an object from the point $(1, 1)$ to $(2, 4)$.

3) Evaluate the line integral

$$\oint_C (y + e^x) dx + (2x + \cos(y^2)) dy$$

where C is the boundary of the region enclosed by the parabolas $y = x^2$ and $x = y^2$, oriented counterclockwise.

4) Find the work done by the vector field

$$\vec{F}(x, y, z) = (x y^2 z^2, x^2 y z^2, x^2 y^2 z)$$

to move a particle along the curve

$$\vec{r}(t) = (\cos^2(t), \sin^2(t), t^3), t \in [0, \pi].$$

5)

(a) Prove that if $\vec{F} = (P, Q, R)$ is a smooth vector field, then $\operatorname{div}(\operatorname{curl}(\vec{F})) = 0$.

(b) Does there exist a vector field \vec{G} on \mathbb{R}^3 such that

$$\operatorname{curl}(\vec{G}) = (x \sin(y), \cos(y), z - xy)?$$

Please justify your answer.