

Math 391 Test 4A

May 13, 2015

Name: _____

Note that both sides of each page may have printed material.

Instructions:

1. Read the instructions.
2. Don't panic! I repeat, do NOT panic!
3. Complete all problems. In this exam, each non-bonus problem is worth 25 points. The weight of the bonus problems are indicated.
4. Show ALL your work to receive full credit. You will get 0 credit for simply writing down the answers (unless it's a case of fill in the blank or state a definition, etc.)
5. Write neatly so that I am able to follow your sequence of steps and box your answers.
6. Read through the exam and complete the problems that are easy (for you) first!
7. No scrap paper, calculators, notes or other outside aids allowed—including divine intervention, telepathy, knowledge osmosis, the smart kid that may be sitting beside you or that friend you might be thinking of texting.
8. In fact, **cell phones should be out of sight!**
9. Use the correct notation and write what you mean! x^2 and $x2$ are not the same thing, for example, and I will grade accordingly.
10. Do NOT commit any of the blasphemies or mistakes I mentioned in the syllabus. I will actually mete out punishment in the way I said I would. I wasn't kidding.
11. Other than that, have fun and good luck!

Remember: Don't take life too seriously. You'll never get out of it alive.

1. (a) State the formula that gives the Laplace Transform $F(s)$ of a function $f(t)$.

(b) Use the definition of the Laplace Transform of a function to compute the Laplace Transform of the function $f(t) = te^t$.

2. Use Laplace transforms to solve the following initial value problem:

$$y'' - y' - 6y = 0, \quad y(0) = 1, y'(0) = -1$$

3. Find the Fourier series for the following function:

$$f(x) = \begin{cases} \pi^2 & 0 \leq x < \pi \\ 0 & -\pi \leq x < 0 \end{cases}$$

4. (a) Find the sine series for the function $f(x) = \pi$ on $[0, \pi]$.

(b) Using your answer in part (a), find the series solution $u(x, t)$ to the partial differential equation for $x \in (0, \pi)$ and $t > 0$:

$$\left\{ \begin{array}{l} u_t = e u_{xx} \\ u(0, t) = u(\pi, t) = 0, t > 0 \\ u(x, 0) = \pi, 0 < x < \pi \end{array} \right. \quad \begin{array}{l} B.C. \\ I.C. \end{array}$$

Bonus Problems:

1. (4 points each) Find the inverse Laplace transform $y(t)$ of the given functions:

(a) $F(s) = \frac{2s+1}{s^2-2s+2}$

(b) $F(s) = \frac{2s-3}{s^2-4}$

2. (4 points) Use separation of variables to write the replace the given PDE with two ODEs:

$$u_{xx} + u_{xt} + u_t = 0$$

3. (3 points) Redo problem 2 using another method.

4. (3 points) Find b so that the given equation becomes exact: $(ye^{2xy} + x) + bxe^{2xy}y' = 0$

5. (6 points) Solve via variation of parameters: $y'' + 4y' + 4y = \frac{e^{-2t}}{1+t}$

TABLE 6.2.1 Elementary Laplace Transforms

| $f(t) = \mathcal{L}^{-1}\{F(s)\}$ | $F(s) = \mathcal{L}\{f(t)\}$ | Notes |
|---|---|----------------------|
| 1. 1 | $\frac{1}{s}, \quad s > 0$ | Sec. 6.1; Ex. 4 |
| 2. e^{at} | $\frac{1}{s-a}, \quad s > a$ | Sec. 6.1; Ex. 5 |
| 3. $t^n, \quad n = \text{positive integer}$ | $\frac{n!}{s^{n+1}}, \quad s > 0$ | Sec. 6.1; Prob. 31 |
| 4. $t^p, \quad p > -1$ | $\frac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$ | Sec. 6.1; Prob. 31 |
| 5. $\sin at$ | $\frac{a}{s^2 + a^2}, \quad s > 0$ | Sec. 6.1; Ex. 7 |
| 6. $\cos at$ | $\frac{s}{s^2 + a^2}, \quad s > 0$ | Sec. 6.1; Prob. 6 |
| 7. $\sinh at$ | $\frac{a}{s^2 - a^2}, \quad s > a $ | Sec. 6.1; Prob. 8 |
| 8. $\cosh at$ | $\frac{s}{s^2 - a^2}, \quad s > a $ | Sec. 6.1; Prob. 7 |
| 9. $e^{at} \sin bt$ | $\frac{b}{(s-a)^2 + b^2}, \quad s > a$ | Sec. 6.1; Prob. 13 |
| 10. $e^{at} \cos bt$ | $\frac{s-a}{(s-a)^2 + b^2}, \quad s > a$ | Sec. 6.1; Prob. 14 |
| 11. $t^n e^{at}, \quad n = \text{positive integer}$ | $\frac{n!}{(s-a)^{n+1}}, \quad s > a$ | Sec. 6.1; Prob. 18 |
| 12. $u_c(t)$ | $\frac{e^{-cs}}{s}, \quad s > 0$ | Sec. 6.3 |
| 13. $u_c(t)f(t-c)$ | $e^{-cs}F(s)$ | Sec. 6.3 |
| 14. $e^{ct}f(t)$ | $F(s-c)$ | Sec. 6.3 |
| 15. $f(ct)$ | $\frac{1}{c}F\left(\frac{s}{c}\right), \quad c > 0$ | Sec. 6.3; Prob. 25 |
| 16. $\int_0^t f(t-\tau)g(\tau) d\tau$ | $F(s)G(s)$ | Sec. 6.6 |
| 17. $\delta(t-c)$ | e^{-cs} | Sec. 6.5 |
| 18. $f^{(n)}(t)$ | $s^n F(s) - s^{n-1}f(0) - \dots - f^{(n-1)}(0)$ | Sec. 6.2; Cor. 6.2.2 |
| 19. $(-t)^n f(t)$ | $F^{(n)}(s)$ | Sec. 6.2; Prob. 29 |