Math 391 Test 1A
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Note that both sides of each page may have printed material.

Instructions:

1. Read the instructions.
2. Panic!!! Kidding, don't panic! I repeat, do NOT panic!
3. Complete all problems. In this exam, each non-bonus problem is worth 20 points. The weight of the bonus problems are indicated.
4. Show ALL your work to receive full credit. You will get 0 credit for simply writing down the answers (unless it's a case of fill in the blank or state a definition, etc.)
5. Write neatly so that I am able to follow your sequence of steps and box your answers.
6. Read through the exam and complete the problems that are easy (for you) first!
7. No scrap paper, calculators, notes or other outside aids allowed—including divine intervention, telepathy, knowledge osmosis, the smart kid that may be sitting beside you or that friend you might be thinking of texting.
8. In fact, **cell phones should be out of sight!**
9. Use the correct notation and write what you mean! x^2 and $x2$ are not the same thing, for example, and I will grade accordingly.
10. Do NOT commit any of the blasphemies or mistakes I mentioned in the syllabus. I will actually mete out punishment in the way I said I would. I wasn't kidding.
11. Other than that, have fun and good luck!

Remember: math is fun, math is beautiful, this test is *not* hard, there is no spoon.

1.

(a) Write down the standard form for a first order linear ODE: $y' + p(t)y = g(t)$

(b) Write down a general form of an exact equation: $M(x,y)\,dx + N(x,y)\,dy = 0$

(c) What condition on the above equation would make it exact? $M_y = N_x$

(d) Let $Q(t)$ be the amount of a pollutant in a tank at time t, let Q_0 be the initial amount of pollutant in the tank, c_{in} be the concentration of pollutant flowing into the tank at a rate r_{in}, and let V_0 be the initial volume in the tank. Let r_{out} be the rate at which the mixture in the tank flows out. Write down the differential equation with initial condition to describe $Q(t)$.

ODE: $Q' = c_{in} r_{in} - \frac{Q}{V_0 + (r_{in} - r_{out})t} \cdot r_{out}$, initial condition: $Q(0) = Q_0$

(e) Write down a general form for a separable first order ODE: $f(y)\,dy = g(t)\,dt$

(f) What condition would make $\frac{dy}{dx} = f(x, y)$ homogeneous? $f(kx, ky) = f(x,y)$ or $f(x,y) = g\left(\frac{y}{x}\right)$ for some function g.

(g) Find the general solution of the following:

(i) $\frac{dy}{dx} = \frac{1 - 2xy - x^2 y}{x^2}$, $x > 0$

(This is linear!)

$\Rightarrow y' + \left(1 + \frac{2}{x}\right)y = \frac{1}{x^2}$

$\mu = e^{\int 1 + \frac{2}{x} dx} = e^{x + \ln|x|^2} = x^2 e^x$

$\Rightarrow \int [x^2 e^x y]' = \int e^x$

$\Rightarrow x^2 e^x y = e^x + C$

$\Rightarrow \boxed{y = \frac{1}{x^2} + \frac{C}{x^2 e^x}}$

(ii) $x\,dx + y e^{-x}\,dy = 0$

(This is separable!)

$y e^{-x} dy = -x\,dx$

$\Rightarrow \int y\,dy = \int -x e^x\,dx$

$\Rightarrow \frac{y^2}{2} = -x e^x + e^x + C$

$\Rightarrow \boxed{y^2 = -2x e^x + 2e^x + C}$