Math 391 Mock Test 2

December 8, 2019

Name: ____

Note that both sides of each page may have printed material.

Instructions:

1. Read the instructions.

2. Don't panic! I repeat, do NOT panic!

3. This test should take you **1 hour and 30 minutes**. This is 15 minutes longer than your actual test will be. The actual test will have a couple less parts.

4. Complete all problems. Bonus problems are optional and will only be counted if all other problems are attempted.

5. Show ALL your work to receive full credit. You will get 0 credit for simply writing down the answers (unless it's a case of fill in the blank or state a definition, etc.) Solutions with no indicated answer, or where several contradictory answers are highlighted will be considered incorrect.

6. Write neatly so that I am able to follow your sequence of steps and box your answers.

7. Read through the exam and complete the problems that are easy (for you) first!

8. No scrap paper, calculators, notes or other outside aids allowed—including divine intervention, telepathy, knowledge osmosis, the smart kid that may be sitting beside you or that friend you might be thinking of texting.

9. In fact, **cell phones should be out of sight!**

10. Use the correct notation and write what you mean! x^2 and $x2$ are not the same thing, for example, and I will grade accordingly.

11. Do NOT commit any of the blasphemies or mistakes I mentioned in the syllabus. I will actually mete out punishment in the way I said I would. I wasn't kidding.

12. Other than that, have fun and good luck!

Remember: Find your math zen. There is nothing but form and algorithm.

1.

(a) Solve: $y^{(4)} + 81y = 0$

(b) Solve: $y^{(6)} - 81y'' = 0$

(b) (ii) Given $y^{(6)} - 81y'' = \pi + 2e^{-3t} + e\sin(3t)$. Determine the suitable form for the function $Y(t)$ with the fewest terms to obtain a particular solution, via the *method of undetermined coefficients*. You need not solve for the coefficients.

2. Solve the following ODEs, assume $x > 0$:
   (a) $x^2y'' + 5xy' + 8y = 0$

   (b) $x^2y'' + 4xy' + 2y = 0$

   (c) $x^2y'' - 3xy' + 4y = 0$

3. Find and classify (as regular or irregular) all the singular points of

$$x^2(1-x)y'' + 2y' + 4y = 0$$

4. For the differential equation:

$$(2 - x^2)y'' + (1 + x)y = 0$$

Compute the recursion formula for the coefficients of the power series solution centered at $x_0 = 0$, and use it to compute the first four nonzero terms of the solution with $y(0) = 2$ and $y'(0) = -1$. Though not required, you may assume $a_n = 0$ for $n < 0$ to avoid some computation.

5. Use Laplace transforms to solve the following initial value problem:

$$y'' + 3y' + 2y = 0, \qquad y(0) = 1, y'(0) = 0$$

6. (a) State the formula that gives the Laplace Transform $F(s)$ of a function $f(t)$.

(b) Use the definition of the Laplace Transform of a function to compute the Laplace Transform of the function $f(t) = te^t$.

7. Find the Fourier series for the following function:

$$f(x) = \begin{cases} \pi^3 & 0 \leq x < \pi \\ 0 & -\pi \leq x < 0 \end{cases}$$

Bonus Problems:

1. Let $f(x)$ be the function in problem 7. Find the series solution $u(x,t)$ to the partial differential equation for $x \in (0,\pi)$ and $t > 0$:

$$\begin{cases} u_t = e u_{xx} & \\ u(0,t) = u(\pi,t) = 0, t > 0 & B.C. \\ u(x,0) = f(x), \ \ 0 < x < \pi & I.C. \end{cases}$$

2. (4 points each) Find the inverse Laplace transform $y(t)$ of the given functions:

   (a) $F(s) = \frac{2s+1}{s^2-2s+2}$ (b) $F(s) = \frac{2s-3}{s^2-4}$

3. (4 points) Use separation of variables to write the replace the given PDE with two ODEs:

$$u_{xx} + u_{xt} + u_t = 0$$