Math 391 Quiz 1
June 9, 2020

23 + 2 Bonus pts.

Name: ANSWERS

Instructions: No calculators! Answer all problems in the space provided.

1. Separable or not? ("Y" or "N"):

$\frac{dy}{dx}=\frac{y+1}{x-5}$: Y $\frac{dy}{dx}=xy+x$: Y $\frac{dy}{dx}=e^x+y$: N $\frac{dy}{dx}=y(y+3)$: Y $\frac{dy}{dx}=\frac{x-1}{y}$: Y

$\frac{dy}{dx}=x+2y$: N $tdt+ye^{-t}dy=0$: Y $y^2(1-x)^{\frac{1}{2}}dy=\arccos x\,dx$: Y

2. Linear or not? ("Y" or "N"):

$(1+y^2)\frac{d^2y}{dt^2}+t\frac{dy}{dt}+y=e^t$: N $y''+\sin(t+y)=\sin t$: N $x^2y''+xy'+2y=\cos x$: Y

3. What is the standard form of a first order linear ODE? : $y'+p(t)y=g(t)$

4. For the ODE above, what is the formula for its integrating factor? $\mu(t)=e^{\int p(t)dt}$ (equation)

5. Separate the variables. (Do not solve the ODEs!):

$\frac{dr}{d\theta}=\frac{r^2}{2\theta}$: $\frac{dr}{r^2}=\frac{d\theta}{2\theta}$ $y'=\frac{2x}{y+x^2y}$: $ydy=\frac{2x}{1+x^2}dx$ $\frac{dy}{dt}=\frac{ty(3-y)}{1+t}$: $\frac{dy}{y(3-y)}=\frac{t}{1+t}dt$

$\frac{dy}{dt}=tye^{3t+y^2}$: $\frac{dy}{ye^{y^2}}=te^{3t}dt$ $dy=(x^2y^2+x^2-y^2-1)dx$: $\frac{dy}{y^2+1}=(x^2-1)\,dx$

6. Solve the following ODEs:

(a) $\frac{dy}{dx}=2y+1$: $y=\frac{Ce^{2x}-1}{2}$ (b) $\frac{dy}{dx}=\frac{3y}{x-1}, y(0)=3$: $y=-3(x-1)^3$

7. If it is assumed that interest is compounded continuously, the Harvesting Model also describes the growth of money in an account. A man puts some money in a bank account earning 3% interest, compounded continuously, and makes withdrawals of \$600, every ~~month~~ year. Suppose he puts P_0 dollars into the account initially. Assume the function $P(t)$ describes the current balance in the account. Describe $P(t)$ using:

An ODE $P'=0.03P-600$, the initial condition for the ODE $P(0)=P_0$

8. Solve the ODE above. Your answer should include the P_0: $P(t)=20000+(P_0-20000)e^{0.03t}$

Bonus problems:

1. Solve the ODEs:

(a) $\frac{dy}{dx}=\frac{x^2+xy+y^2}{x^2}$: $y=x\tan(\ln|x|+C)$

(b) $2xy-x^2+(2y+x^2+1)\frac{dy}{dx}=0$ *Soln:* $x^2y-\frac{x^3}{3}+y^2+y=C$