Math 346 Study Guide - Spring, 2024

(1) Matrix Arithmetic - Be able to perform matrix computations. Know when the sum or product of two matrices can be defined. Know the two interpretations of $AX = B$ for an $m \times n$ matrix A. (m equations in n unknowns; linear combination of the columns of A)

(2) Matrix Inverse and Transpose - know the definition and properties.
(a) If a matrix A has a row of zeroes, can it have an inverse? Explain.
(b) If A and B are invertible $m \times m$ matrices, prove that AB is invertible.
(c) When is a matrix symmetric? For any matrix A show that AA^T is defined and is symmetric.

(3) Echelon and Reduced Echelon Form - Know the meaning and be able to perform the steps to put a matrix in echelon form. Be able to compute the rank of a matrix.

(4) Elementary Matrices - Know the definition of the elementary matrices associated with a row operation.
Know that doing a row operation is the same as multiplying by the corresponding elementary matrix.
Explain why an elementary matrix is invertible.
If A is an $m \times m$ matrix row equivalent to Q in reduced echelon explain what is true about Q when the rank equals m or when the rank is less than m. In any case, explain why $Q = UA$ with U an invertible matrix.

(5) Solve the system $AX = B$ by using the augmented matrix $(A|B)$.
Explain why a system can have either 0 or 1 or infinitely many solutions. When do the different possibilities occur?

(6) Compute the inverse of A by using $(A|I)$ and compute the inverse directly if A is a 2×2 matrix.
Using the inverse of A, when it exists, solve the system $AX = B$.

(7) Linear Transformation - When is a function $T : \mathbb{R}^n \to R^m$ a linear transformation?
Explain why $T_A(X) = AX$ is such a linear transformation.
Using the standard basis $\{\mathbf{e}_1, \ldots, \mathbf{e}_m\}$ for $\mathbb{R}^m$ show that any linear transformation $T : \mathbb{R}^n \to R^m$ comes from a matrix A in this way.

(8) Determinants - know the properties and computation using reduction to echelon form.

Part II

(1) Subspace of a Vector Space - know the definition and compute examples.

(2) For a list $\{v_1, v_2, \ldots, v_n\}$ in a vector space V know the definition and important properties of the following:

(a) linear combination.
(b) linear independence and linear dependence.
(c) the span of a list.
(d) when the list spans, i.e. spans the entire space, or when

a list spans a subspace.

(e) basis of a space or subspace.

(f) dimension of a space or subspace.

(g) coordinates of a vector with respect to a basis.

(3) For an $m \times n$ matrix A be able to define and compute a basis for:

(a) The Null Space of A (the solution space of the homogeneous system).

(b) The Column Space.

(c) The Row Space.

(4) For an $m \times n$ matrix A row equivalent to Q in Reduced Echelon Form, understand the following equivalences:

(a)The columns of A form a linearly independent list.

(b) Every column of Q has a leading 1.

(c) The rank of A equals n.

and understand the following equivalences:

(a) The columns of A span $\mathbb{R}^m$.

(b) Every row of Q has a leading 1.

(c) The rank of A equals m.

and when $m = n$ so that A is a square matrix understand the following equivalences:

(a) A is invertible.
(b) $Q = I$.
(c) The rank of A equals n.
(d) The columns form a basis for $\mathbb{R}^n$.