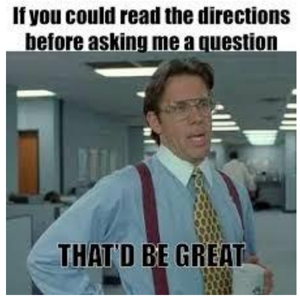
Math 346 Test 2
July 19, 2018

Name: ____

Note that both sides of each page may have printed material.



Instructions:

1. Read the instructions.

2. Panic!!! Kidding, don't panic! I repeat, do NOT panic!

3. Complete all problems in the actual test. Fully justify! Bonus problems are optional, and will only be counted if all parts of all other problems are attempted.

4. Show ALL your work to receive full credit. You will get 0 credit for simply writing down the answers.

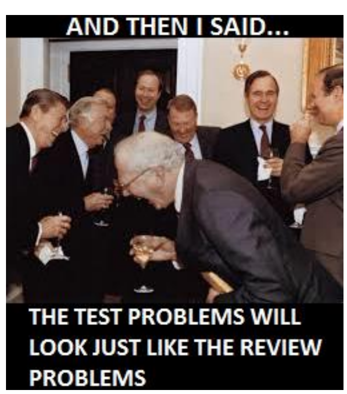
5. Write neatly so that I am able to follow your sequence of steps and box your answers.

6. Read through the exam and complete the problems that are easy (for you) first!

7. Scientific calculators are allowed but not required. **Graphing calculators are strictly forbidden!** You are also NOT allowed to use notes, or other aids—including, but not limited to, divine intervention/inspiration, the internet, telepathy, knowledge osmosis, the internet, the smart kid that may be sitting beside you or that friend you might be thinking of texting. In fact, **cell phones should be out of sight!**

8. Use the correct notation and write what you mean! x^2 and $x2$ are not the same thing, for example, and I will grade accordingly.

9. Other than that, have fun and good luck!



1. Let $D: P_2 \to P_2$ be the differentiation operator $D(\boldsymbol{p}) = p'(x)$.

(a) (10 points) Find $[D]_B$, where $B = \{2, 2 - 3x, 2 - 3x + 8x^2\}$

(b) (5 points) Use part (a) to compute $[D(6 - 6x + 24x^2)]_B$.

(c) (5 points) Compute $D(6 - 6x + 24x^2)$ using the above.

2. (a) (5 points) Prove that $W = \{A \in M_{22} : A^T = A\}$ is a subspace of M_{22}.

(b) (10 points) Let $A_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, $A_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ and $A_3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. Prove that $B = \{A_1, A_2, A_3\}$ is a basis for W defined above.

3. (a) (5 points) Prove that if $T_1: U \to V$ and $T_2: V \to W$ are linear transformations of vector spaces, then the composition $T_2 \circ T_1: U \to W$ is also a linear transformation.

(b) (5 points) A linear transformation is called an *isomorphism* if it is one-to-one and onto. If there is an isomorphism between two vector spaces, then the vector spaces are said to be *isomorphic*. Prove that the property of being isomorphic is *transitive*. That is, prove that if U, V and W are vector spaces, and U is isomorphic to V and V is isomorphic to W, then U is isomorphic to W.

4. Let $A = \begin{pmatrix} 1 & 2 & 3 & 3 & 0 \\ 2 & 4 & 7 & 7 & 0 \\ 3 & 6 & 9 & 9 & -1 \\ 1 & 2 & 4 & 4 & 1 \end{pmatrix}$

(a) (8 points) Find a basis for the column space of A

(b) (6 points) Find a basis for the null space of A

(c) (4 points) Find $rank(A)$ and $Nullity(A)$.

(d) (2 points) The correct answer to (c) is an example of a general principle. What is this principle?

5. (a) (5 points) Let S be a finite set of linearly independent vectors. Prove that any non-empty subset of S is also a linearly independent set. Hint: a proof by contradiction is a good way to go.

(b) (5 points) Suppose $S = \{\vec{u}, \vec{v}\}$ is a linearly independent set. Prove that $B = \{\vec{u} + \vec{v}, \vec{u} - \vec{v}\}$ is also linearly independent.

6. (a) (10 points) Suppose $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation obtained by first reflecting over the x-axis, then rotating counter-clockwise by an angle of $\frac{\pi}{2}$. Find the standard matrix for T. Is T one-to-one? Justify.

(b) (10 points) Determine if the following linear operator $T: \mathbb{R}^3 \to \mathbb{R}^3$ is one-to-one, and whether T^{-1} exists. T is defined via

$$\begin{aligned} w_1 &= 2x_1 + 2x_2 + x_3 \\ w_2 &= 2x_1 + x_2 - x_3 \\ w_3 &= 3x_1 + 2x_2 + x_3 \end{aligned}$$

Bonus Problems: You must attempt all parts of all other problems to be eligible.

1. Consider the matrix $A = \begin{pmatrix} 3 & 4 \\ -1 & -2 \end{pmatrix}$.
   (a) (10 points) Find the eigenvalues and corresponding eigenvectors of A.

   (b) (2 points) Find an invertible matrix P and a diagonal matrix D such that $D = P^{-1}AP$.

   (c) (5 points) Compute A^5, write as a 2x2 matrix.

(d) (3 points) Solve the following system for the functions $y_1(t)$ and $y_2(t)$, subject to the initial conditions $y_1(0) = 1$ and $y_2(0) = 2$.

$$\begin{Bmatrix} y_1{}'(t) & = & 3y_1(t) + 4y_2(t) \\ y_2{}'(t) & = & -y_1(t) - 2y_2(t) \end{Bmatrix}$$

?? Linear independence, span, basis, null space, subspace, vector space, does this set span that one, linear operator,...????

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