Math 346 Test 1
June 28, 2018

Name: ____

Note that both sides of each page may have printed material.

Instructions:

1. Read the instructions.

2. Panic!!! Kidding, don't panic! I repeat, do NOT panic!

3. Complete all problems in the actual test. Bonus problems are, of course, optional. And they will only be counted if all other problems are attempted.

4. Show ALL your work to receive full credit. You will get 0 credit for simply writing down the answers.

5. Write neatly so that I am able to follow your sequence of steps and box your answers.

6. Read through the exam and complete the problems that are easy (for you) first!

7. Scientific calculators are allowed, but not required. **Graphing calculators are strictly forbidden!** You are also NOT allowed to use notes, or other aids—including, but not limited to, divine intervention/inspiration, the internet, telepathy, knowledge osmosis, the smart kid that may be sitting beside you or that friend you might be thinking of texting.

8. In fact, **cell phones should be out of sight!**

9. Use the correct notation and write what you mean! x^2 and $x2$ are not the same thing, for example, and I will grade accordingly.

10. Other than that, have fun and good luck!

    May the force be with you. But you can't ask it to help you with your test.

1. Let $A = \begin{pmatrix} 2 & 2 & -2 \\ -1 & 0 & 2 \\ 1 & 2 & 1 \end{pmatrix}$.

(a) (15 points) Find A^{-1}

(b) (5 points) Use A^{-1} to solve the system $\begin{cases} 2x & + & 2y & - & 2z & = & 1 \\ -x & & & + & 2z & = & 0 \\ x & + & 2y & + & z & = & 1 \end{cases}$

2. Let $A = \begin{pmatrix} 3 & 0 & -3 & 0 \\ 0 & -1 & -1 & 0 \\ 1 & 0 & 0 & 1 \\ -1 & 0 & 1 & 1 \end{pmatrix}$

(a) (12 points) Find $\det A$

(b) (2 points) Find $\det(A^{-3})$

(c) (2 points) Find $\det(A^{-1})$

(d) (4 points) Find $\det(2A^3A^{-1}A^TA^{-2})$

3. (a) (15 points) Solve the system by a method of your choice: $\begin{cases} 3x + 2y - z = 0 \\ x + y = -1 \\ x - z = 2 \end{cases}$

Write your solution in vector form.

(b) (5 points) Could the above system be solved using Cramer's rule? Explain.

4. (a) Prove or disprove – Assuming the matrices shown have dimensions so that the operations are defined. (2 points each):

(i) If A and B are invertible, then so is AB

(ii) If A and B are invertible, then so is $A - B$

(iii) If $A^T A = A$, then A^T must be I_n

(iv) If A is invertible, then $\det(A^{-1}BA) = \det(B)$

(v) If B is a square matrix, then BB^T is symmetric

(vi) If B is a square matrix, then $B + B^T$ is symmetric

(vii) If $A^T A = A$, then A is symmetric, and $A = A^2$

5. (a) (8 points) Prove that a square matrix A is invertible if and only if $\det A \neq 0$.

(b) (8 points) Prove that a square matrix A is invertible if and only if $A^T A$ is invertible.

6. (10 points) Consider the system in problem 1(b), that is, the system

$$\begin{cases} 2x & + & 2y & - & 2z & = & 1 \\ -x & & & + & 2z & = & 0 \\ x & + & 2y & + & z & = & 1 \end{cases}$$

Use Cramer's Rule to solve for y only. (Do NOT solve for x or z!) No credit will be given for any other method.

Bonus Problems:

Definition: Vectors $\overrightarrow{v_1}, \overrightarrow{v_2}, \dots, \overrightarrow{v_n}$ are said to be linearly independent if the ONLY solution to the equation

$$c_1\overrightarrow{v_1} + c_2\overrightarrow{v_2} + \cdots + c_n\overrightarrow{v_n} = \vec{0}$$

is the trivial solution, $c_1 = c_2 = \cdots = c_n = 0$.

Definition: A set B of vectors form a basis for a vector space if the set of vectors is (1) linearly independent and (2) they span the vector space—that is, every vector in the vector space can be expressed as a linear combination of vectors in B.

1. (5 points) Show that the functions x^2 and x^3 are linearly independent.

2. (5 points) Show that the set $B = \{\vec{\imath}, \vec{\jmath}, \vec{k}\} = \{< 1,0,0 >, < 0,1,0 >, < 0,0,1 >\}$ is a basis for $\mathbb{R}^3$

3. (5 points) Prove that, in a vector space, $(-1)\vec{u} = -\vec{u}$.

4. (5 points) Prove that, in a vector space, $0\vec{u} = \vec{0}$


GOT 120?
CLOSE ENOUGH.