City College **Department of Mathematics**

Math 346 **Final Exam** **Spring 2023**

Do 10 complete problems. They are worth 10 points each. You receive credit only for 10 problems.

1. (a) For the matrix

$$A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & -2 \\ 2 & 1 & 3 \end{pmatrix}$$

compute the inverse, A^{-1}.

(b) Suppose you know that, for a matrix B, one has $B^{-1} = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & -1 \\ 1 & 2 & 2 \end{pmatrix}$.

What is the solution of the following system of linear equations:

$$B\mathbf{x} = \begin{pmatrix} 2 \\ 3 \\ -2 \end{pmatrix} \quad ?$$

2. For the matrix

$$A = \begin{pmatrix} 1 & -2 & 0 & 1 & 0 \\ -2 & 4 & 1 & 1 & 0 \\ 0 & 0 & -1 & -3 & 1 \\ 1 & -2 & 1 & 4 & 1 \end{pmatrix}$$

compute a basis for **(a)** the row space, **(b)** the column space, and **(c)** the nullspace. For each space, compute also the dimension.

3. For the matrix

$$A = \begin{pmatrix} 0 & -2 & -2 \\ 2 & 4 & 2 \\ -2 & -2 & 0 \end{pmatrix}$$

compute the eigenvalues and for each eigenvalue compute a basis for the associated eigenspace.

4. For each of the following subsets of the vector space of 2×2 matrices, determine whether or not it is a subspace. **Explain** your answer.

(a) The set of matrices of the form $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ such that $a + d \geq 0$.

(b) The set of all invertible 2×2 matrices.

5. Let $\{v_1, v_2, v_3, v_4\}$ be a basis for a vector space V. Answer the following questions and justify your answers.

(a) Is $\{v_1, v_1 + v_2, v_1 + v_3, v_1 + v_4\}$ a basis for V?

(b) Is $\{v_1 + v_2, v_1 + v_3, v_1 + v_4\}$ a basis for V?

(c) Can you find a vector $u \in V$, not equal to v_1, v_2, v_3 or v_4 such that $\{u, v_1, v_2, v_3, v_4\}$ is a basis for V?

6. (a) Find the determinant of the matrix $B = \begin{pmatrix} 1 & 1 & 0 & 3 \\ -2 & 0 & -1 & -4 \\ 1 & 1 & 2 & 4 \\ -1 & -1 & 1 & -4 \end{pmatrix}$.

(b) Solve the system $B\mathbf{x} = \mathbf{0}$, where B is the 4×4 matrix given in part (a).

7. Let $T\colon \mathbb{R}^3 \to \mathbb{R}^2$ be given by $T(x, y, z) = (x + y, x - y + z)$.

(a) Find the matrix of T with respect to the standard bases for $\mathbb{R}^3$ and $\mathbb{R}^2$.

(b) Is T one-to-one? **Explain** your answer.

(c) Is T onto? **Explain** your answer.

8. Find the line of best fit $y = a + bx$ for the data points $(1, 3)$, $(2, 4)$ and $(-1, -1)$.

9. Find the orthogonal projection of the vector $(1, 2, 1, 1)$ onto the subspace of $\mathbb{R}^4$ spanned by $(1, 0, 1, 1), (-1, 1, 1, 0), (0, 1, -1, 1)$.

10. Let P_k denote the vector space of polynomials of degree at most k. Let $T\colon P_3 \to P_2$ be the function defined by $T(p(x)) = p''(x) - 2p'(x)$. This is a linear map.

(a) Find the matrix for T with respect to the standard bases for P_3 and P_2.

(b) Find a basis for $\ker T$.