Do 10 complete problems. They are worth 10 points each. You receive credit only for 10 problems.
1. (a) For the matrix

\[ A = \begin{pmatrix} 
1 & 2 & 0 \\
0 & 3 & -2 \\
2 & 1 & 3 
\end{pmatrix} \]

compute the inverse, \( A^{-1} \).

(b) Suppose you know that, for a matrix \( B \), one has \( B^{-1} = \begin{pmatrix} 
1 & 1 & 2 \\
0 & 1 & -1 \\
1 & 2 & 2 
\end{pmatrix} \).

What is the solution of the following system of linear equations:

\[ Bx = \begin{pmatrix} 
2 \\
3 \\
-2 
\end{pmatrix} \]
2. For the matrix

\[ A = \begin{pmatrix}
1 & -2 & 0 & 1 & 0 \\
-2 & 4 & 1 & 1 & 0 \\
0 & 0 & -1 & -3 & 1 \\
1 & -2 & 1 & 4 & 1
\end{pmatrix} \]

compute a basis for (a) the row space, (b) the column space, and (c) the nullspace. For each space, compute also the dimension.
3. For the matrix

\[ A = \begin{pmatrix} 0 & -2 & -2 \\ 2 & 4 & 2 \\ -2 & -2 & 0 \end{pmatrix} \]

compute the eigenvalues and for each eigenvalue compute a basis for the associated eigenspace.
4. For each of the following subsets of the vector space of $2 \times 2$ matrices, determine whether or not it is a subspace. Explain your answer.

(a) The set of matrices of the form \[
\begin{pmatrix}
  a & b \\
  c & d 
\end{pmatrix}
\] such that $a + d \geq 0$.

(b) The set of all invertible $2 \times 2$ matrices.
5. Let \{v_1, v_2, v_3, v_4\} be a basis for a vector space \(V\). Answer the following questions and justify your answers.

(a) Is \{v_1, v_1 + v_2, v_1 + v_3, v_1 + v_4\} a basis for \(V\)?

(b) Is \{v_1 + v_2, v_1 + v_3, v_1 + v_4\} a basis for \(V\)?

(c) Can you find a vector \(u \in V\), not equal to \(v_1, v_2, v_3\) or \(v_4\) such that \{\(u, v_1, v_2, v_3, v_4\)\} is a basis for \(V\)?
6. (a) Find the determinant of the matrix $B = \begin{pmatrix} 1 & 1 & 0 & 3 \\ -2 & 0 & -1 & -4 \\ 1 & 1 & 2 & 4 \\ -1 & -1 & 1 & -4 \end{pmatrix}$.

(b) Solve the system $Bx = 0$, where $B$ is the $4 \times 4$ matrix given in part (a).
7. Let $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ be given by $T(x, y, z) = (x + y, x - y + z)$.

(a) Find the matrix of $T$ with respect to the standard bases for $\mathbb{R}^3$ and $\mathbb{R}^2$.

(b) Is $T$ one-to-one? Explain your answer.

(c) Is $T$ onto? Explain your answer.
8. Find the line of best fit $y = a + bx$ for the data points $(1, 3)$, $(2, 4)$ and $(-1, -1)$. 
9. Find the orthogonal projection of the vector \((1, 2, 1, 1)\) onto the subspace of \(\mathbb{R}^4\) spanned by \((1, 0, 1, 1), (-1, 1, 1, 0), (0, 1, -1, 1)\).
10. Let $P_k$ denote the vector space of polynomials of degree at most $k$. Let $T: P_3 \to P_2$ be the function defined by $T(p(x)) = p''(x) - 2p'(x)$. This is a linear map.

(a) Find the matrix for $T$ with respect to the standard bases for $P_3$ and $P_2$.

(b) Find a basis for $\ker T$. 