MATH 21200 SYLLABUS AND HOMEWORK

Suggested Periods	Section	Topics	Exercises
1	5.5	Substitution Review	3, 11, 17, 21, 25, 29, 37, 47, 55, 61
1	5.6	Definite Integral Substitutions	3, 7, 9, 11 , 29, 31, 33, 37 (Hint:Let $u=\sin\theta$), 41
1	7.1	The Logarithm Defined as an Integral	3, 5, 7, 11 (Hint:$2\sqrt{x}+2x=2\sqrt{x}(1+2\sqrt{x})$), 19, 23, 31, 37*
2	7.2	Exponential Change and Separable Differential Equations	1, 5, 9, 15, 25, 27, 29, 37, 41, s1
1	7.3	Hyberbolic Functions	5, 13, s1, s2, s3
1	8.1	Using Basic Integration Formulas	1, 2, (Hint:$x^2=x^2+1-1$) ,3 (Hint: $\sec^2x=1+\tan^2x$), 5,9 (Hint:Let $u=e^z$), 15, 33 (Hint: Multiply top & bottom by 1+y), s1, s2
2	8.2	Integration by Parts	5, 8, 13, 17, 21, 25, 27, 29, 31, 33, 39, 47,
1.5	8.3	Trigonometrtic Integrals	7*, 11, 13*, 17, 19, 35, 37, 41, 53(51)
2	8.4	Trigonometric Substitutions	1, 5, 9, 11, 21(17), 27(23), 35(31), 39(35), 43
2.5	8.5	Integration of Rational Functions by Partial Fractions	11, 15*, 35, 41, s1
1	8.7	Numerical Integration (omit error estimates)	1(a), 2(a), 5, s1
2.5	8.8	Improper Integrals	5*, 17, 19, 21, 27, 29*, 37*, 41*, 51, 55, 57, 59, 65
1	10.01	Sequences	9, 15, 17, 23, 35, 37, 45, 51, 53, 63, 67, 123
2	10.02	Infinite Series	1, 3, 5, 7, 9, 13, 33, 35, 41, 53, 57, 65
2.5	10.03	The Integral Test	3, 6, 11, 15, 17, 23, 27, 28*, 37, 55
2.5	10.04	Comparison Tests	1, 5, 9, 13, 15, 17, 19, 21, 25, 26, 35, 47
2	10.05	Absolute Convergence; The Ratio and Root Tests	1, 5, 9, 13, 15, 29, 35, 42, 43, 67, 70
2	10.06	Alternating Series and Conditional Convergence	5, 7, 9, 11, 15, 17*, 22, 23, 25, 27, 31, 39, 51, 53
2.5	10.07	Power Series	5, 9, 11, 15, 17, 25, 27, 41*, 53, s1
2	10.08	Taylor and Maclaurin Series	1, 3, 5, 11, 13, 15, 19, 25, 35
1	10.09	Convergence of Taylor Series (omit Theorem 24)	1, 7 (Hint: Use series ln(1+x)), 10, 13, 21, s1
2	10.10	The Binomial Series and Applications of Taylor Series (Cover Evaluating Non-Elementary Integrals only)	23, s1
2.5	11.3	Polar Coordinates	1, 3, 5, 7, 11, 13, 27, 45* (answer should include the origin), 53, 55, 57
1	11.5	Area in Polar Coordinates (omit Length)	1, 3, 6, 9*, 11, 15
2.5	12.1	Three-Dimensional Coordinate Systems	1, 3, 7, 11, 17, 21, 27*, 31(a), 35(a), 37(a), 59, s1, s2
5	12.6	Cylinders and Quadric Surfaces	1, 3, 5*, 6, 9, 11, 13, 17, 23, 25, 29, 35, 37, 41, s1, s2

Total: 49 hours.

SUPPLEMENTARY HOMEWORK PROBLEMS

7.2/s1. In a certain region, the population, $P(t)$, in thousands of people, t years after census there began, is approximated using an exponential growth model. The initial census showed a population of $P_0 = 90$, and the population two years later was $P(2) = 120$.

(a) Find a formula for $P(t)$.

(b) Find the population after 4 years. Simplify the answer, which is an integer.

(c) Find the population after 5 years. (The answer is not an integer.)

(d) How long does it take for the population to double?

(e) How long (i.e., how many years) will it take for the population to reach a million people?

7.3/s1. Find $\cosh x$ when $\sinh x = 2$.

7.3/s2. Find $\dfrac{d}{dx}[x^2 \ln(\sinh(5x))]x$.

7.3/s3. Evaluate: $\displaystyle\int x^2 \cosh(x^3)\, dx$

8.1/s1. Evaluate: $\displaystyle\int_1^2 2^{3x}\, dx$

8.1/s2. $\displaystyle\int_0^{\sqrt{\pi/4}} 4x \tan(x^2)\, dx$

8.1/s3. Evaluate: $\displaystyle\int \frac{\sec(\ln x)}{x}\, dx$

10.7/s1. Find the interval of convergence: $\sum_{n=0}^{\infty} \frac{(x+3)^n}{(n+1)^2)3^n}$

Remember to check the endpoints if applicable.

10.7/s2. Find the interval of convergence: $\sum_{n=0}^{\infty} \frac{(x-4)^n}{\ln(n+2)}$

Remember to check the endpoints if applicable.

10.10/s1. (a) Find the first five terms of the Maclaurin series (i.e., the power series centered at zero) for $f(x) = \frac{1}{2x+1}$.

(b) Find the first four nonzero terms for the derivative, $f'(x)$, of $f(x)$.

(c) Use the answer to (b) to approximate $f'(.05)$, with an error not to exceed .01, and verify that your answer has the required accuracy.

11.3/s1. Graph the polar equation $r = 3 - 2\cos\theta$, and label all x- and y-intercepts, if any exist.

11.3/s2. Graph the polar equation $r = 3 + 6\sin\theta$, and label all x- y-intercepts, if any exist.

12.6/s1. Graph and label all vertices, if any exist:

$$x^2 + 4y^2 - z^2 - 6x + 4z + 9 = 0$$

12.6/s2. (a) Graph and label all vertices, if any exist:

$$2x^2 + 27z^2 + 4x - 16 = 0$$

scroll down for answers and selected solutions

ANSWERS AND SELECTED SOLUTIONS TO EVEN NUMBERED AND SUPPLEMENTARY HOMEWORK PROBLEMS

7.2/s1. In a certain region, the population, $P(t)$, in thousands of people, t years after census there began, is approximated using an exponential growth model. The initial census showed a population of $P_0 = 90$, and the population two years later was $P(2) = 120$.

(a) Find a formula for $P(t)$.

(b) Find the population after 4 years. Simplify the answer, which is an integer.

(c) Find the population after 5 years. (The answer is not an integer.)

(d) How long does it take for the population to double?

(e) How long (i.e., how many years) will it take for the population to reach a million people?

(a) $P(t) = P_0\left(\frac{P(2)}{P(0)}\right)^{t/(2-0)} = 90\left(\frac{120}{90}\right)^{t/2} = 90\left(\frac{4}{3}\right)^{t/2}$ or $P(t) = 90e^{(t/2)\ln(4/3)}$.

(b) $P(4) = 90(4/3)^{4/2} = 160$ or $P(4) = 90e^{[ln(4/3)]2} = 90e^{ln(16/9)} = 90(16/9) = 160$.

(c) $P(5) = 90(4/3)^{5/2} = 90(32/(9\sqrt{3}) = 320/\sqrt{3}$.

(d) Let t_d be the number of years it takes the population to double. $(1/2)P_0 = P_0(4/3)^{t_d/2}$. Equate the log of both sides: $\ln(1/2) = (t_d/2)\ln(4/3)$;

$t_d = 2\ln(1/2)/\ln(4/3) = -\ln 4/\ln(4/3)$.

(e) One million is 1000 thousands. $1000 = 90(4/3)^{t/2}$; $100/9 = (4/3)^{t/2}$; $t = 2\ln(100/9)/\ln(4/3)$.

7.3/s1. $\sqrt{3}$ (and $x > 0$)

7.3/s2. $2x\ln(\sinh(5x)) + 5x^2\coth(5x)$

7.3/s3. $\frac{1}{3}\sinh(x^3)\,dx$

8.1/2. $\int \frac{x^2}{x^2+1} = \frac{(x^2+1)-1}{x^2+1}\,dx = x - \arctan x + C$

8.1/s1. $\frac{56}{3\ln 2}$

8.1/s2. $\ln 2$

8.1/s3. $\ln|\tan(\ln x) + \sec(\ln x)| + C$

8.2/8. $\left(\frac{x}{3} - \frac{1}{9}\right)e^{3x} + C$

10.1/123. Is the sequence $a_n = \frac{2^n 3^n}{n!}$ monotone or bounded?

$a_n = 6^n/n!$ and, for $n \geq 6$,

$$a_{n+1} = \left(\frac{6}{1}\frac{6}{2}\cdots\frac{6}{6}\right)\left(\frac{6}{7}\cdots\frac{6}{n}\right)\frac{6}{n+1} = \frac{a_n}{n+1} \leq a_n,$$

and direct computation shows $a_n < a_{n+1}$ for $n < 4$. Thus the sequence in not monotone (the answer in the text is wrong). The sequence is bounded: $0 \leq a_n \leq a_6$ for all n.

Note: A sequence a_n, is called *eventually* monotone if there is some integer N such that a_n for $n \geq N$ is monotone. For most of what is done in Chapter 10, eventually monotone is as useful as monotone. The sequence above is eventually decreasing.

10.3/6. Use the Integral Test for the series $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$.

Let $u = x^2$. $\int_2^{\infty} \frac{1}{(x \ln x)^2}\,dx = \int_{\ln 2}^{\infty} \frac{1}{u^2}\,du$ which is convergent, since 2¿1. Thus the series is convergent.

10.3/28. Is the series $\sum_{n=1}^{\infty}(1+\frac{1}{n})^n$ convergent or divergent?

Each term a_n of the series is greater than or equal to 1, so $\lim a_n$ is not 0, and the series is divergent by the Test for Divergence.

10.4/26. Convergent by direct comparison with $b_n = \dfrac{1}{n^{3/2}}$.

10.5/42. Divergent by Ratio Test.

10.5/70. Is the series $\displaystyle\sum_{n=1}^{\infty} \frac{2^{n^2}}{n!}$ convergent or divergent?

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{2^{(n+1)^2}}{(n+1)!}\frac{n!}{2^{n^2}} = \frac{2^{n^2+2n+1}}{(n+1)!}\frac{n!}{2^{n^2}} = \frac{2^{2n+1}}{n+1} \longrightarrow \infty,$$

so $L = \infty$ and the series diverges by the Ratio Test.

10.6/22. Absolutely convergent by direct comparison of $|a_n|$ with $b_n = \dfrac{1}{n^2}$.

10.7/s1. $[-6, 0]$

10.7/s2. Find the interval of convergence: $\displaystyle\sum_{n=0}^{\infty} \frac{(x-4)^n}{\ln(n+2)}$

$$L = \lim \frac{\ln(n+2)}{\ln(n+3)} = \lim \frac{\ln(x+2)}{\ln(x+3)} = \lim \frac{1/(x+2)}{1/(x+3)} = 1.$$

$R = 1/L = 1$. The interval of convergence is $(3, 5)$.

10.10/s1. (a) Find the first five terms of the Maclaurin series (i.e., the power series centered at zero) for $f(x) = \dfrac{1}{2x+1}$.

(b) Find the first four nonzero terms for the derivative, $f'(x)$, of $f(x)$.

(c) Use the answer to (b) to approximate $f'(.05)$, with an error not to exceed .01, and verify that your answer has the required accuracy.

(a) Write $f(x)$ in the form $f(x) = \dfrac{1}{1-(-2x)}$ and substitute into the equation $\dfrac{1}{1-x} = \sum_{n=0}^{\infty} x^n$:

$$\begin{aligned} f(x) &= \sum_{n=0}^{\infty}(-2x)^n \\ &= \sum_{n=0}^{\infty}(-1)^n 2^n x^n \\ &= 1 - 2x + 2^2x^2 - 2^3x^3 + 2^4x^4 \pm \ldots \end{aligned}$$

$$\begin{aligned} \text{(b)} f'(x) &= \sum_{n=0}^{\infty}(-1)^n 2^n n x^{n-1} \\ &= -2 + 2^2 \cdot 2x - 2^3 \cdot 3x^2 + 2^4 \cdot 4x^3 \pm \ldots \\ &= -2 + 8x^2 - 24x^3 + 64x^3 \pm \ldots \end{aligned}$$

$$\begin{aligned} \text{(c)} f'\left(\frac{1}{20}\right) &= -2 + \frac{8}{400} - \frac{24}{8000} + \frac{64}{160000} \pm \\ &= \boxed{-2 + \frac{1}{50}} - \frac{3}{1000} \pm \ldots \\ &\approx -2 + \frac{1}{50} \qquad \text{Answer} \end{aligned}$$

$$R_3 \le \frac{24}{8000} = \frac{3}{1000} \le \frac{1}{100} = .01$$

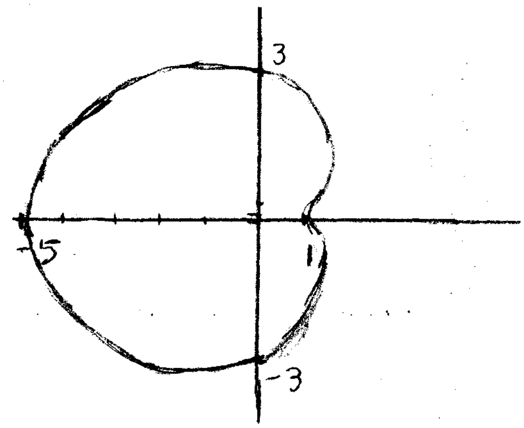
11.3/s1. Graph the polar equation $r = 3 - 2\cos\theta$, and label all x- and y-intercepts, if any exist.

$r(0) = 1$

$r(\pi/2) = 3$

$r(\pi) = 5$

$r(3\pi/2) = 3$



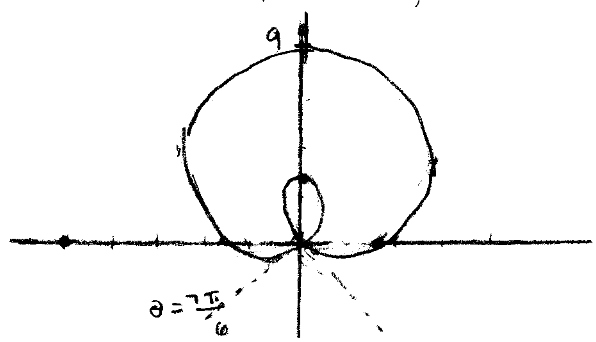
11.3/s2. Graph the polar equation $r = 3 + 6\sin\theta$, and label all y- intercepts, if any exist.

$r(0) = 3$

$r(\pi/2) = 9$

$r(\pi) = 3$

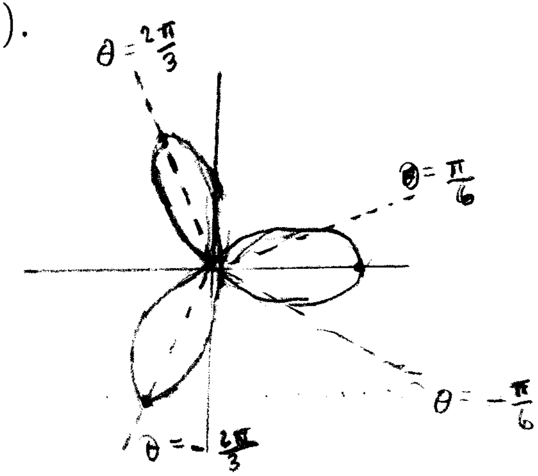
$r(3\pi/2) = -3$



$$0 = r(0) = 3 + 6\sin\theta \text{ if } \theta = \frac{7\pi}{6}, -\frac{\pi}{6}$$

11.5/6. Find the area inside one loop the rose leaf curve $r = \cos 3\theta$ (graph below was given).

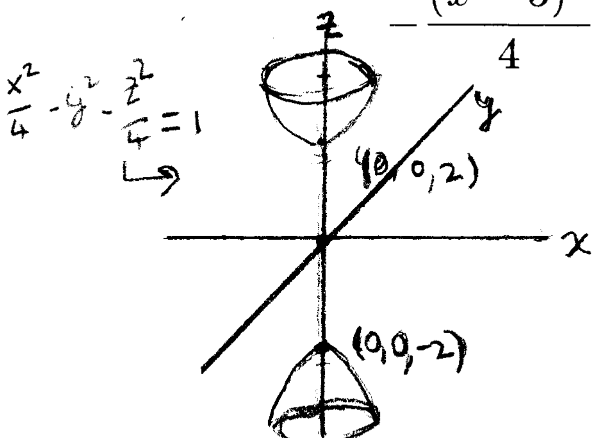
$r = \cos 3\theta = 0$ if $\theta = \pm\frac{\pi}{6}$.

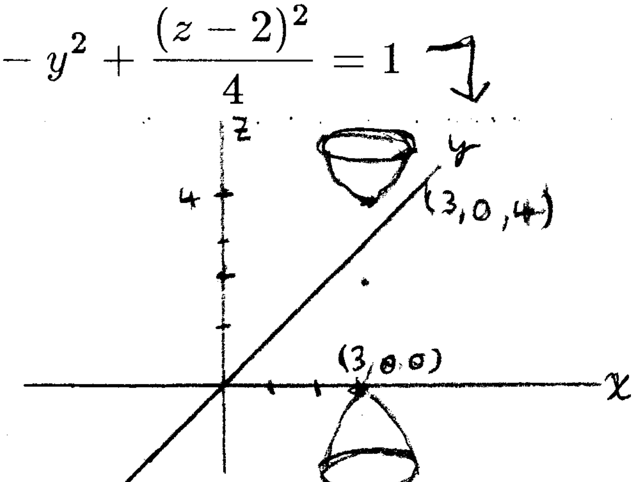


$$A = \frac{1}{2}\int_{-\pi/6}^{\pi/6} \cos^2 3\theta\, d\theta = \int_{-\pi/6}^{\pi/6} \frac{1}{4} + \frac{1}{4}\cos 6\theta\, d\theta = \frac{1}{4}\frac{\pi}{3} = \frac{\pi}{12}.$$

12.6/s1. Graph and label all vertices, if any exist:
$x^2 + 4y^2 - z^2 - 6x + 4z + 9 = 0$

$$x^2 - 6x + 4y^2 - [z^2 - 4z] + 9 = 0$$
$$(x-3)^2 - 9 + 4y^2 - [(z-2)^2 - 4] + 9 = 0$$
$$(x-3)^2 + 4y^2 - (z-2)^2 = -4$$
$$-\frac{(x-3)^2}{4} - y^2 + \frac{(z-2)^2}{4} = 1$$





12.6/s2. (a) Graph and label all vertices, if any exist:
$2x^2 + 27z^2 + 4x - 16 = 0$

$$\frac{(x+1)^2}{9} + \frac{3z^2}{2} = 1$$
$$\frac{(x+1)^2}{9} + \frac{3z^2}{2} = 1$$
$$\frac{(x+1)^2}{9} + \frac{z^2}{2/3} = 1$$