MATH 212 SAMPLE IN-PERSON FINAL EXAM

Answer ALL questions (10 points each). Show all work.

1. (a) Evaluate: $\int_1^2 \frac{x^2+x+1}{x^3+x^2}\,dx$

(b) Evaluate: $\int e^x \sin x\,dx$

2. (a) Evaluate: $\int \sin^2(3x)\cos^3(3x)\,dx$

(b) Evaluate: $\int_0^{\pi/6} \cos^2(3x)\,dx$

(c) Find the Trapezoidal Rule estimate of $\int_0^2 x^4\,dx$ obtained by breaking the interval of integration into $n = 4$ subintervals.

3. (a) Evaluate: $\int \frac{x^3}{\sqrt{9-4x^2}}\,dx$

(b) Find the value of each convergent improper integral and show why each divergent improper integral is divergent.

(i) $\int_2^\infty \frac{2+e^{-x}}{x+2}\,dx$ (ii) $\int_2^4 \frac{1}{(x-2)^2}\,dx$ (iii) $\int_2^\infty e^{2-x}\,dx$

4. State, for each series, whether it converges absolutely, converges conditionally or diverges. Name a test which supports each conclusion and show the work to apply the test.

(a) $\sum_{n=0}^\infty \frac{(-1)^n n}{3n+1}$ (b) $\sum_{n=0}^\infty \frac{(-1)^n 5^n}{3^{2n}}$ (c) $\sum_{n=0}^\infty \frac{(-1)^n}{3n+1}$

5. (a) Find the interval of convergence of the series

$\sum_{n=0}^\infty \frac{(x-2)^n}{(n+2)3^n}$.

Remember to check the endpoints, if applicable.

(b) Graph the equation $x^2 + 2x + 9y^2 + 9z^2 = 8$, labelling the coordinates of the center and one vertex if any exist.

6. (a) Let $f(x) = \dfrac{1}{1+2x}$. (i) Find the first five terms of the Maclaurin series (i.e., the series centered at 0) representation of $f(x)$.

(ii) Use the result in (i) to find $f'(.01)$ with an error less than or equal .001. Justify that your answer has the required accuracy.

(b) Find the limit or show it does not exist:

$\lim_{(x,y)\to(0,0)} \dfrac{x^2+y^4}{x^4+y^2}$

7. (a) Find an equation of the plane containing the points $(1,0,-1)$, $(2,-1,0)$ and $(1,2,3)$.

(b) Find parametric equations for the line through $(5,8,0)$ and parallel to the line through $(4,1,-3)$ and $(2,0,2)$.

(c) Is the vector $\mathbf{v}$ parallel, perpendicular or neither to the plane $z = x + 2y$, where

(i) $\mathbf{v} = \langle 2,0,2\rangle$ and (ii) $\mathbf{v} = \langle 1,2,1\rangle$?

8. (a) Find parametric equations for the line of intersection of the planes $2x + 3y + z = 1$ and $x - 3y + 2z = 2$.

(b) Find the distance between the planes $x + 2y + 2z = 2$ and $x + 2y + 2z = 4$.

9. (a) Find the rectangular coordinates of the point(s) of the graph of the polar equation $r = 4\sin\theta$ that are farthest from the y-axis.

(b) Sketch the graph of the parametric equations $x = 3\cos t$, $y = 4\sin t$, labelling all intercepts.

10. (a) Find $\frac{\partial f}{\partial x}$, for $f(x,y) = \log_2(3^{x^2} + y)$

(b) For a positive integer n, find $\dfrac{\partial^n f}{\partial z^n}$, the n-th partial derivative of f with respect to z, for $f(x,y,z) = e^{x+y^2+2z}$

(c) Find f_{xy} for $f(x,y) = xy^3 + \frac{x^2+3x+1}{\sqrt{4+x^2}}$