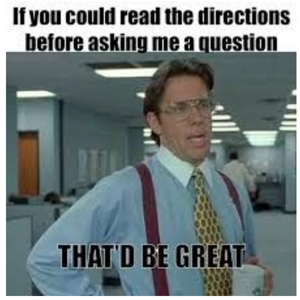
Math 212 RS2 Test 2

Quarantine, day 42.

Name: ____

Note that both sides of each page may have printed material.



Instructions:

1. Read the instructions.
2. Panic!!! Kidding, don't panic! I repeat, **do NOT panic!**
3. Complete all problems in the actual test. Bonus problems are, of course, optional, and will only be counted if all other problems are attempted.
4. **You have 90 minutes to complete the test.**
5. Show ALL your work to receive full credit. You will get 0 credit for simply writing down the answers (unless it's a case of fill in the blank or state a definition, etc.)
6. Write neatly so that I am able to follow your sequence of steps and box, or otherwise indicate, your answers. Solutions with no indicated answer or several contradictory answers will be considered incorrect.
7. Read through the exam and complete the problems that are easy (for you) first!
8. You are NOT allowed to use notes, calculators, or other aids—including, but not limited to, divine intervention/inspiration, the internet, telepathy, knowledge osmosis, the smart kid that may be sitting beside you or that friend you might be thinking of texting.
9. In fact, **cell phones should be out of sight! If you are caught with a cellphone you will be asked to leave the exam and you'll be given a zero. That goes for smart watches too!**
10. Use the correct notation and write what you mean! x^2 and $x2$ are not the same thing, for example, and I will grade accordingly.

1. (a) (4 points) Convert $(x, y) = (-4,4)$ to polar coordinates (r, θ)

(b) (4 points) Convert $(r, \theta) = \left(-2, \frac{2\pi}{3}\right)$ to rectangular coordinates (x, y)

(c) (4 points) Describe the following region in 3-space (including its boundary) using polar coordinate inequalities with an inequality for z. Include a sketch. *The region in the first octant, above the paraboloid* $z = x^2 + y^2$ *and below the plane* $z = 4$.

2. Consider the curve C given parametrically by $x = 1 + \sqrt{t},\ \ y = e^{t^2},\quad 0 < t < \infty$.
   (a) (3 points) Compute $\frac{dy}{dx}$

   (b) (6 points) Find an equation for the tangent line to C at $t = 1$.

   (c) (3 points) Set up, but do not compute, an integral to find the arc length of the part of C on the interval $0 \leq t \leq 1$.

3. (5 points each) Draw rough sketches of the following.

(a) $y^2 - x^2 = 1$

(b) $z = 1 - \sqrt{x^2 + y^2}$

(c) $z = x^2 - y^2$

(d) $x^2 + y^2 + \frac{z^2}{9} = 1$

4. (5 points each)

   (a) Find the equation of the plane through the point $(5,3,5)$ that is orthogonal to the vector $2\vec{\boldsymbol{i}} + 3\vec{\boldsymbol{j}} - \vec{\boldsymbol{k}}$.

   (b) Find the equation of the line through $(-6,2,3)$ that is parallel to the line $x = 3 + 2t$, $y = 1 + t$, $z = 1 - t$.

   (c) What is the angle between the vectors $< -1,-1,2 >$ and $< 3,1,1 >$.

5. (6 points) Find the perpendicular distance from the point $(0,2,3)$ to the line $x = 3 + 2t$, $y = 1 + t$, $z = -1 + 2t$.

6. (5 points each) Determine whether or not the following limits exist, justify your claim.

$$(a) \quad \lim_{(x,y)\to(2,2)} \frac{x-y}{x^4-y^4}$$

$$(b) \quad \lim_{(x,y)\to(0,0)} \frac{1-\cos(xy)}{xy}$$

$$(c) \quad \lim_{(x,y,z)\to(0,0,0)} \frac{xy+yz}{x^2+y^2+z^2}$$

$$(d) \quad \lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}}$$

7. (5 points each) Find the indicated partial derivatives:

(a) f_{xyy} given that $f(x,y) = x^y$

(b) $\frac{\partial^3 V}{\partial r \partial s \partial t}$ given that $V = \ln(r + s^2 + t^3)$

(c) $f_y\left(1, \frac{1}{2}\right)$ given that $f(x,y) = y \sin^{-1}(xy)$

Bonus: Bonus problems will only be counted if all non-bonus problems are attempted.

1. (3 points each) Let $\vec{a} = < -1, -1, 2 >$ and $\vec{b} = < 2, 2, -1 >$.
   (a) Compute $\vec{a} \times \vec{b}$.

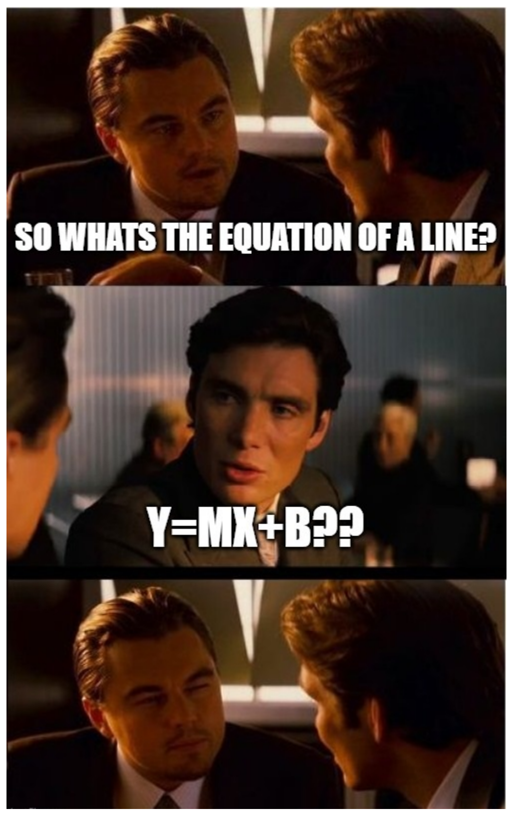
   (b) Find the area of the parallelogram formed by $\vec{a}$ and $\vec{b}$.

   (c) Find the smallest angle between $\vec{a}$ and $\vec{b}$. You may leave your answer in terms of an inverse trig function.

2. (3 points) Sketch the surface $f(x, y) = \ln y$

3. (4 points) Find and sketch the domain of $f(x,y) = \frac{\sqrt{x - y^2}}{1 - y^2}$

4. (4 points) Sketch a contour map of $f(x,y) = 9x^2 + y^2$ showing several level curves.


SO WHATS THE EQUATION OF A LINE?
Y=MX+B??