Math 201 Test 2 Review 2

December 5, 2019

Name: SOLUTIONS

Note that both sides of each page may have printed material.

Instructions:

1. Read the instructions.
2. Panic!!! Kidding, don't panic! I repeat, **do NOT panic**! Don't look down, while you're at it.
3. Complete all problems in the actual test. Bonus problems are, of course, optional. And they will only be counted if all other problems are attempted.
4. **You have 1 hour to complete the test.**
5. Show ALL your work to receive full credit. You will get 0 credit for simply writing down the answers.
6. Write neatly so that I am able to follow your sequence of steps and box your answers.
7. Read through the exam and complete the problems that are easy (for you) first!
8. Scientific calculators are allowed, but you are NOT allowed to use notes, graphing calculators, or other aids—including, but not limited to, divine intervention/inspiration, the internet, telepathy, knowledge osmosis, the smart kid that may be sitting beside you or that friend you might be thinking of texting.
9. In fact, **cell phones should be out of sight!**
10. Use the correct notation and write what you mean! x^2 and $x2$ are not the same thing, for example, and I will grade accordingly.
11. Other than that, have fun and good luck!

    May the force be with you. But you can't ask it to help you with your test.

1. Compute the following integrals (5 points each):

(a) $\int \frac{\sqrt{x}-2x^2}{x}\,dx = \int x^{-1/2} - 2x\,dx$

$= \boxed{2x^{1/2} - x^2 + C}$

(b) $\int (2x+1)(x^2+1)^2\,dx = \int (2x+1)(x^4+2x^2+1)\,dx$

$= \int 2x^5 + 4x^3 + 2x + x^4 + 2x^2 + 1\,dx$

$= \boxed{\frac{1}{3}x^6 + x^4 + x^2 + \frac{1}{5}x^5 + \frac{2}{3}x^3 + x + C}$

(c) $\int \sec^2\theta \tan\theta\,d\theta$

$u = \tan\theta$

$du = \sec^2\theta\,d\theta$

$\Rightarrow \int u\,du$

$= \frac{u^2}{2} + C$

$= \boxed{\frac{\tan^2\theta}{2} + C}$

OR

$\int \sec\theta \sec\theta \tan\theta\,d\theta$

$u = \sec\theta$

$du = \sec\theta\tan\theta\,d\theta$

$\Rightarrow \int u\,du$

$= \frac{u^2}{2} + C$

$= \boxed{\frac{\sec^2\theta}{2} + C}$

$$(d)\int_2^{e^2} \frac{1}{x\ln\sqrt{x}}\,dx = \int_2^{e^2} \frac{2}{x\ln x}\,dx$$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$\Rightarrow x\,du = dx$

When $x = e^2$

$\Rightarrow u = \ln e^2 = 2$

when $x = 2$

$\Rightarrow u = \ln 2$

$$\int_{\ln 2}^{2} \frac{2}{xu}\cdot x\,du$$

$$= 2\int_{\ln 2}^{2} \frac{1}{u}\,du$$

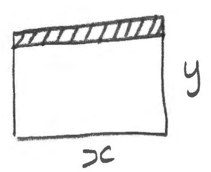
$$= 2\ln|u|\Big|_{\ln 2}^{2}$$

$$= \boxed{2\ln|2| - 2\ln|\ln 2|}$$

2. (15 points) The manager of a department store wants to build a 600 square-foot rectangular enclosure in the store's parking lot to display some equipment. Three sides of the enclosure will be built of redwood fencing, costing \$14 per running foot, while the fourth side will be built of cement blocks costing \$28 per running foot. Find the dimensions of the enclosure that will minimize the cost of materials.

① Read!

② Diagram



③ Constraint

$xy = 600$

Objective

$C = 14(x+2y) + 28x$

$\Rightarrow C = 42x + 28y$

④ New objective

$y = \frac{600}{x}$

$\Rightarrow C = 42x + \frac{28(600)}{x}$

⑤ Minimize Cost

$C' = 42 - \frac{28(600)}{x^2} = 0$ or $\cancel{\text{und}}$

$\Rightarrow 42x^2 = 28(600)$

$\Rightarrow x^2 = \frac{\cancel{28}^{\,24}(\cancel{600}^{\,200})}{\cancel{42}\ \cancel{14}\ \cancel{2}}$

$\Rightarrow x^2 = 400$

$\Rightarrow x = 20$

$\Rightarrow y = \frac{60\cancel{0}}{2\cancel{0}} = 30$

⑥ Dimensions

$\boxed{30 \times 20 \text{ ft enclosure}}$

where the cement side is a 20-ft side.

3. (a) (10 points) A group of angry calculus students (allegedly) throw Jhevon off a 48-foot cliff with an initial velocity of 32 ft/s. What is Jhevon's position function? Hint: the acceleration due to gravity is $-32\ ft/s^2$.

$$a(t) = -32$$

$$\Rightarrow v(t) = \int -32\,dt = -32t + C$$

$$v(0) = 32 \Rightarrow -32(0) + C = 32$$

$$\Rightarrow C = 32$$

$$\Rightarrow v(t) = -32t + 32$$

$$\Rightarrow s(t) = \int -32t + 32\,dt$$

$$= -16t^2 + 32t + C$$

$$s(0) = 48 \Rightarrow -16(0)^2 + 32(0) + C = 48$$

$$\Rightarrow C = 48$$

$$\boxed{s(t) = -16t^2 + 32t + 48}$$

(b) (10 points) Compute $\displaystyle \frac{d}{dx}\int_{2x}^{\sqrt{x}} \frac{\cos\theta}{\theta}\,d\theta$ $\displaystyle = \frac{d}{dx}\left(\int_c^{\sqrt{x}} \frac{\cos\theta}{\theta}\,d\theta - \int_c^{2x} \frac{\cos\theta}{\theta}\,d\theta\right)$

$$= \frac{\cos\sqrt{x}}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} - \frac{\cos(2x)}{2x} \cdot 2$$

$$= \boxed{\frac{\cos\sqrt{x}}{2x} - \frac{\cos(2x)}{x}}$$

4. (5 points) Use linear approximation to approximate $\sqrt{16.01}$. Write your answer as a fraction.

Set $f(x) = \sqrt{x}$

$$\Rightarrow f'(x) = \frac{1}{2\sqrt{x}}$$

Set $x = 16.01$ and $a = 16$.

Using $f(x) \approx f(a) + f'(a)(x-a)$

$$\Rightarrow \sqrt{16.01} = f(16.01) \approx f(16) + f'(16)(16.01 - 16)$$

$$= \sqrt{16} + \frac{1}{2\sqrt{16}}(0.01)$$

$$= 4 + \frac{1}{8} \cdot \frac{1}{100}$$

$$= 4 + \frac{1}{800} = \boxed{\frac{3201}{800}}$$

5. (a) (10 points) Find the value(s) of c guaranteed by the Mean Value Theorem for the function $f(x) = \sqrt[3]{x}$ on the interval $[0,1]$.

What conditions allowed you to apply the Mean Value Theorem here?

We can apply the MVT since f is continuous on $[0,1]$ and differentiable on $(0,1)$.

$\Rightarrow \exists\, c \in (0,1)$ so that

$$\Rightarrow f'(c) = f_{avg}$$

$$\Rightarrow f'(c) = \frac{f(1)-f(0)}{1-0}$$

$$\Rightarrow \frac{1}{3}c^{-2/3} = 1$$

$$\Rightarrow c^{2/3} = \frac{1}{3}$$

$$\Rightarrow c = \pm\sqrt{\frac{1}{27}} \Rightarrow \boxed{c = \frac{1}{\sqrt{27}}}$$

(since $-\frac{1}{\sqrt{27}} \notin (0,1)$).

(b) (10 points) Find the absolute extrema of the function $f(x) = 24x - 2x^3$ on $[0,3]$.

① Crit pts

$f' = 24 - 6x^2 = 0$ or $\cancel{\text{und}}$ for crit pts

$\Rightarrow x^2 = 4$

$\Rightarrow x = \pm 2$

Take $x = 2$. ($-2 \notin [0,3]$)

$f(2) = 48 - 16 = 32$

② Test the end points

$f(0) = 0$

$$\begin{aligned} f(3) &= 24(3) - 2(3)^3 \\ &= 8(3)^2 - 2(3)^3 \\ &= 2(3)^2[4-3] \\ &= 18 \end{aligned}$$

③ Compare

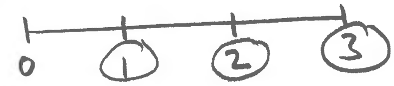
$f(0) = 0 \rightarrow$ abs min

$f(2) = 32 \rightarrow$ abs max

6. Consider the function $f(x) = x^2 + 1$ on the interval $[0,3]$.

(a) (10 points) Use a finite Riemann sum with three equal subintervals and right-hand endpoints to approximate the area under $f(x)$ on the interval.

$$\Delta x = \frac{b-a}{n} = \frac{3-0}{3} = 1$$



$$A \approx R_3 = \left(f(1) + f(2) + f(3)\right)\Delta x$$
$$= \left[(1)^2+1 + (2)^2+1 + (3)^2+1\right](1)$$
$$= \boxed{17}$$

(b) (10 points) Compute the exact area under $f(x)$ on the interval by using the limit at infinity of a Riemann sum.

$$\Delta x = \frac{b-a}{n} = \frac{3-0}{n} = \frac{3}{n}$$

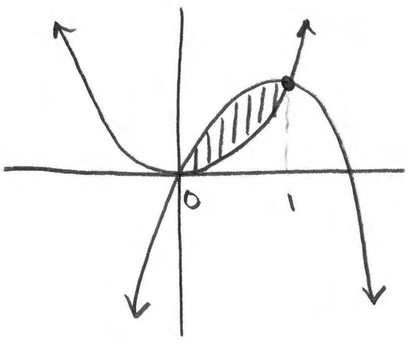
$$x_i = a + i\Delta x = 0 + \frac{3i}{n} = \frac{3i}{n}$$

$$\Rightarrow A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$
$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(\frac{3i}{n}\right)^2 + 1\right]\left(\frac{3}{n}\right)$$
$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\left(\frac{27}{n^3}\right)i^2 + \left(\frac{3}{n}\right)\right)$$
$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{27}{n^3} i^2 + \frac{3}{n}\right)$$
$$= \lim_{n\to\infty} \left[\frac{27}{n^3} \sum_{i=1}^{n} i^2 + \frac{3}{n} \sum_{i=1}^{n} 1\right]$$

$$= \lim_{n\to\infty} \left[\frac{27}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} + \frac{3}{n} \cdot n\right]$$
$$= \frac{27(2)}{6} + 3$$
$$= 9 + 3$$
$$= \boxed{12}$$

Bonus Problems: 5 points each. (You must complete all problems in the actual test to be eligible). Show your work!

1. Compute the area bounded between the curves $y = x^2$ and $y = 2x - x^2$. Include a sketch of the bounded region.



For limits Intersection

$x^2 = 2x - x^2$

$\Rightarrow 2x^2 - 2x = 0$

$\Rightarrow 2x(x-1) = 0$

$x=0, x=1$

$A = \int_0^1 \text{top} - \text{bottom}\, dx$

$= \int_0^1 2x - x^2 - x^2\, dx$

$= x^2 - \frac{2}{3}x^3 \Big|_0^1$

$= 1 - \frac{2}{3}$

$= \boxed{\frac{1}{3}}$

3. (a) (3 points) Compute the average value of $f(x) = x^3$ on the interval $[0,3]$.

$\bar{f} = \frac{1}{b-a}\int_a^b f(x)\,dx$

$= \frac{1}{3}\int_0^3 x^3\,dx$

$= \frac{x^4}{12}\Big|_0^3$

$= \boxed{\frac{27}{4}}$

(b) (2 points) Find $\int_{-\pi/4}^{\pi/4} \frac{t^4 \tan t}{2 + \cos t}\, dt = \boxed{0}$

odd function over a balanced interval.