Math 201 Test 1A
October 30, 2019

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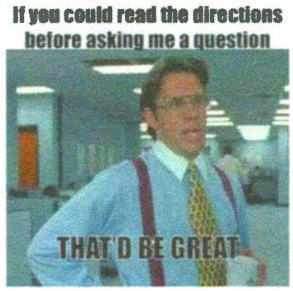
Note that both sides of each page may have printed material.

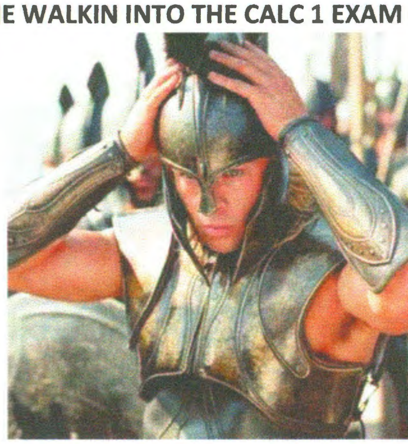


Instructions:

1. Read the instructions.
2. Panic!!! Kidding, don't panic! I repeat, **do NOT panic!**
3. Complete all problems in the actual test. Bonus problems are optional. Point values are indicated.
4. **You have 1 hour and 15 minutes to complete the test.**
5. Show ALL your work to receive full credit. You will get 0 credit for simply writing down the answers (unless it's a case of fill in the blank or state a definition, etc.)
6. Write neatly so that I am able to follow your sequence of steps and box, or otherwise indicate, your answers. Solutions with no indicated answer or several contradictory answers will be considered incorrect.
7. Read through the exam and complete the problems that are easy (for you) first!
8. Don't commit any of the blasphemies mentioned in the syllabus!
9. You are NOT allowed to use notes, calculators, or other aids—including, but not limited to, divine intervention/inspiration, the internet, telepathy, knowledge osmosis, the smart kid that may be sitting beside you or that friend you might be thinking of texting.
10. In fact, **cell phones should be out of sight! If you are caught with a cellphone you will be asked to leave the exam and you'll be given a zero. That goes for smart watches too!**
11. Use the correct notation and write what you mean! x^2 and $x2$ are not the same thing, for example, and I will grade accordingly.

Other than that, have fun and good luck!

ME WALKIN INTO THE CALC 1 EXAM LIKE



1. (a) (15 points) Let $f(x) = 2 - x - x^2$. Use the limit definition of the derivative to find $f'(x)$. **No credit will be given for any other method!**

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{2 - (x+h) - (x+h)^2 - (2 - x - x^2)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{2} - \cancel{x} - h - \cancel{x^2} - 2xh - h^2 - \cancel{2} + \cancel{x} + \cancel{x^2}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(-1 - 2x - h)}{\cancel{h}}$$

$$= \lim_{h \to 0} (-1 - 2x - h)$$

$$\Rightarrow \boxed{f'(x) = -1 - 2x}$$

(b) (5 points) Using your answer to part (a), compute the equation of the tangent line to $f(x)$ at the point where $x = 2$. **Write your line in $y = mx + b$ form.**

When $x = 2$, $y = -4$

also, $m = f'(2) = -1 - 2(2) = -5$

$$\Rightarrow y + 4 = -5(x - 2)$$

$$\Rightarrow \boxed{y = -5x + 6}$$

2. (a) (5 points) Using an equation, define what it means for a function $f(x)$ to be continuous at a point $(a, f(a))$.

$$\boxed{\lim_{x \to a} f(x) = f(a)}$$

(b) (10 points) Consider the function

$$f(x) = \begin{cases} \dfrac{x^2 - 4}{x - 2}, & x < 2 \\ ax^2 - bx + 3, & 2 \le x < 3 \\ 2x - a + b, & x \ge 3 \end{cases}$$

Find the values of a and b that will make the function continuous everywhere.

If $x \neq 2, x \neq 3$, we're fine.

At $x = 2$

We need

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = f(2)$

$\Rightarrow \lim_{x \to 2^-} \left(\frac{x^2 - 4}{x - 2}\right) = \lim_{x \to 2^+} (ax^2 - bx + 3)$

$\Rightarrow 4 = 4a - 2b + 3$

$\Rightarrow 4a - 2b = 1$

At $x = 3$

We need

$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x) = f(3)$

$\Rightarrow \lim_{x \to 3^-} (ax^2 - bx + 3) = \lim_{x \to 3^+} (2x - a + b)$

$\Rightarrow 9a - 3b + 3 = 6 - a + b$

$\Rightarrow 10a - 4b = 3$

Together we need

$4a - 2b = 1$ — ①

$10a - 4b = 3$ — ②

$8a - 4b = 2$ — ①×2

$2a = 1$ — subtract

$\Rightarrow a = 1/2$

$\Rightarrow 4(\tfrac{1}{2}) - 2b = 1$

$\Rightarrow b = 1/2$

$$\Rightarrow \boxed{a = \tfrac{1}{2}, \; b = \tfrac{1}{2}}$$

(c) (5 points) Using interval notation, state where the following function is continuous. Justify your claim!

$$g(x) = \frac{\sqrt{x}}{\sqrt{2x^2 - x - 1}}$$

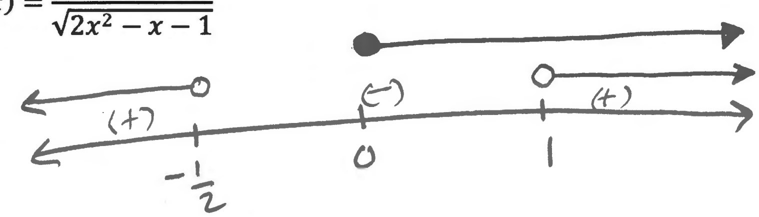
We need:

$x \geq 0$ for top

$2x^2 - x - 1 > 0$ for bottom

$\Rightarrow (2x+1)(x-1) > 0$

$x = -\frac{1}{2}, x = 1$



Take the overlap.

$$\Rightarrow \boxed{g(x) \text{ is continuous on } (1, \infty)}$$

3. (5 points each) Compute the following limits. **Show your work!** Note that $\infty, -\infty$, and DNE are valid answers.

Maximum concentration!!!

(i) $\lim_{x\to-2} \frac{x^2-x-2}{x^2+x-2} \to f(x)$

$= \lim_{x\to-2} \frac{(x-2)(x+1)}{(x+2)(x-1)} \to \lim_{x\to-2^+} f(x) \to \frac{(-4)(-1)}{0^+\cdot -3} = -\infty$

while

$\lim_{x\to-2^-} f(x) \to \frac{(-4)(-1)}{0^-\cdot -3} = +\infty$

$\Rightarrow \boxed{DNE}$

(ii) $\lim_{x\to-3} \frac{x^2-9}{2x^2+7x+3}$

$= \lim_{x\to-3} \frac{(x-3)\cancel{(x+3)}}{(2x+1)\cancel{(x+3)}}$

$= \frac{-6}{-5}$

$= \boxed{\frac{6}{5}}$

(iii) $\lim_{x\to0} \sqrt{x^3+x^2}\sin\frac{\pi}{x} = L$

If $x \neq 0$,

$-1 \le \sin\frac{\pi}{x} \le 1$

$\Rightarrow -\sqrt{x^3+x^2} \le \sqrt{x^3+x^2}\sin\frac{\pi}{x} \le \sqrt{x^3+x^2}$

Taking "$\lim_{x\to0}$" of all sides,

$\Rightarrow 0 \le \lim_{x\to0}\sqrt{x^3+x^2}\sin\frac{\pi}{x} \le 0$

$\Rightarrow \boxed{L = 0 \text{ by the Squeeze Thm}}$

(iv) $\lim_{x\to0} \frac{\sin x}{x+\tan x}$

$= \lim_{x\to0} \frac{\frac{\sin x}{x} \to 1}{\cancel{\frac{x}{x}}^{1} + \frac{\tan x}{x} \to 1}$

$= \boxed{\frac{1}{2}}$

(v) $\lim_{x\to0} \frac{e^x-1-x}{x^2} \to \frac{0}{0}$

$\overset{L'H}{=} \lim_{x\to0} \frac{e^x-1}{2x} \to \frac{0}{0}$

$\overset{L'H}{=} \lim_{x\to0} \frac{e^x}{2}$

$= \boxed{\frac{1}{2}}$

4. **(10 points) Use the intermediate value theorem to show that there is at least one solution to the equation $x^4 + x = 3$ in the interval (1,2). What condition(s) allow you to use the intermediate value theorem here?**

Let $f(x) = x^4 + x - 3$. It suffices to show $f(x) = 0$ for some x in $(1,2)$. $\boxed{\text{Since } f(x) \text{ is continuous, the IVT applies on } (1,2).}$

Note: $f(1) = -1 < 0$ and $f(2) = 15 > 0$. By the IVT, there exists some $x \in (1,2)$ such that $f(x) = 0$. ∎

5. **(5 points each) Compute and simplify $y' = \frac{dy}{dx}$ for the following:**

(a) $y = \tan x + x \ln x - \frac{3}{e^x}$

$$\Rightarrow \boxed{y' = \sec^2 x + \ln x + 1 + 3e^{-x}}$$

(b) $y = \ln \sqrt[3]{\frac{xe^{-x}}{(x-1)^5}} = \frac{1}{3}\left(\ln x - x - 5\ln(x-1)\right)$

$$\Rightarrow \boxed{y' = \frac{1}{3}\left(\frac{1}{x} - 1 - \frac{5}{x-1}\right)}$$

(c) $y = x^{x^2+1}$

$$y = e^{(x^2+1)\ln x}$$

$$\Rightarrow \boxed{y' = x^{x^2+1}\left(2x\ln x + \frac{x^2+1}{x}\right)}$$

(d) $y = \frac{2 - x^2}{5 + 2^x}$

$$\Rightarrow \boxed{y' = \frac{-2x(5+2^x) - (2-x^2)(2^x \ln 2)}{(5+2^x)^2}}$$

(e) $e^2 + 4xy + \ln(xy) = 5y$

$$\Rightarrow e^2 + 4xy + \ln x + \ln y = 5y$$

$$\Rightarrow 4y + 4xy' + \frac{1}{x} + \frac{y'}{y} = 5y'$$

$$\Rightarrow y'\left(\frac{1}{y} + 4x - 5\right) = -4y - \frac{1}{x}$$

$$\Rightarrow y' = \frac{-4y - \frac{1}{x}}{\frac{1}{y} + 4x - 5} \cdot \frac{-xy}{-xy}$$

$$\Rightarrow \boxed{y' = \frac{4xy^2 + y}{5xy - 4x^2y - x}}$$

Bonus Problems: You must attempt all problems in the actual test to be eligible.

1. (5 points) Use the ϵ-δ definition of a limit to show that $\lim_{x\to 1} \frac{x^2-1}{x-1} = 2$.

Let $\varepsilon > 0$ be given. Choose $\delta = \varepsilon$.

Then, if $0 < |x-1| < \delta$, we have

$$\left|\frac{x^2-1}{x-1} - 2\right| = \left|\frac{\cancel{(x-1)}(x+1)}{\cancel{x-1}} - 2\right| = |x-1| < \delta = \varepsilon. \quad \blacksquare$$

2. (10 points) On what interval is the function $f(x) = \frac{x^2}{x^2-4}$ concave up? Concave down? At what x-value(s) does it have an inflection point, if any?

$$f'(x) = \frac{2x(x^2-4) - x^2(2x)}{(x^2-4)^2}$$

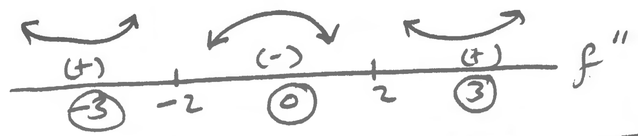
$$= \frac{-8x}{(x^2-4)^2}$$

$$\Rightarrow f'' = \frac{-8(x^2-4)^{\cancel{2}} + 8x \cdot 2\cancel{(x^2-4)}2x}{(x^2-4)^{\cancel{4}3}}$$

$$= \frac{-8x^2+32+32x^2}{(x^2-4)^3}$$

$$= \frac{24x^2+32}{(x^2-4)^3}$$

$$\Rightarrow f'' = \frac{8(3x^2+4)}{(x^2-4)^3}$$

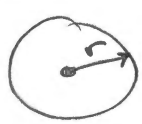


$\boxed{\begin{array}{l} f \text{ is C.U. on } (-\infty,-2)\cup(2,\infty) \\ f \text{ is C.D. on } (-2,2) \\ \text{There are no inflections.} \end{array}}$

3. (5 points) The radius of a circle is increasing at a rate of 2 m/min. How fast is the area changing when the area is 9π cubic meters?

① Read

② Diagram



③ Know $\frac{dr}{dt} = 2$ want $\frac{dA}{dt}$ when $A = 9\pi$

④ Equation

$A = \pi r^2$

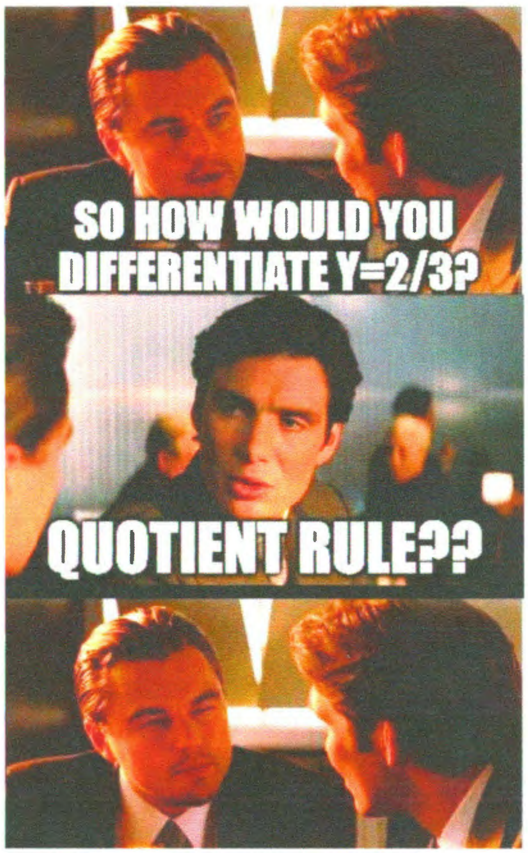
⑤ Differentiate

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

⑥ Plug in

When $A = 9\pi \Rightarrow \pi r^2 = 9\pi \Rightarrow r = 3$

$$\Rightarrow \frac{dA}{dt} = 2\pi(3)(2) = \boxed{12\pi}\ m^2/min$$


SO HOW WOULD YOU DIFFERENTIATE Y=2/3?
QUOTIENT RULE??