THE CITY COLLEGE OF NEW YORK
DEPARTMENT OF MATHEMATICS
SPRING 2023

MATH 21300, FINAL EXAMINATION

Your name (Print): ______________________________
First Last

EMPL. ID: ______________________________

INSTRUCTIONS:

- Notes, books, and calculators are not to be used.
- All work on this exam is to be your own.
- Read each problem carefully. Unless otherwise instructed, be sure to show your work. Remember that it is your obligation to answer each question clearly and in a way that convinces the grader that you understand how to solve the problem.
- Stop working immediately at the end of the exam when time is called.

Problem #	Out of	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
total	100	

1. Let P_1 be the plane $3x - y + z = 5$ and P_2 the plane $x + 2y = -9$.

a) Find the angle between the planes P_1 and P_2. You may use an inverse trigonometric function in your answer.

Angle:

b) Find parametric equations for the line in which P_1 and P_2 intersect.

Line:

2. Find the following limit or show that it does not exist.

$$\lim_{(x,y)\to(-1,1)} \frac{xy - x}{x^2 - y^2}.$$

Answer:

3. Let $f(x, y) = \dfrac{2x}{y+1}$.

a) Find the linearization $L(x, y)$ of the function f at the point $P_0(1, 1)$.

$L(x, y)$:

b) Find an upper bound for the magnitude of the error E in the approximation $f(x, y) \approx L(x, y)$ over the rectangle R: $0.9 \leq x \leq 1.1,\ 0.8 \leq y \leq 1.2$. You may use decimals and fractions in your answer.

$|E| \leq$:

4. Let $f(x, y) = x^2 + y^2 - xy - x$.

a) Find and classify all critical points of f.

Answer:

b) Find the absolute minimum value of the function f on the triangular region bounded by the lines $x = 0$, $y = 1$, and $x - 2y = 0$.

Absolute min.:

5. a) Change the following Cartesian integral into an equivalent polar integral.

$$\int_0^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} e^{x^2+y^2}\, dx dy.$$

Answer:

b) Evaluate the polar integral from part a).

Value:

6. A solid E is bounded below by the surface $z = y^2$, above by the plane $z = 1$, and on the sides by the planes $x = 0$ and $x = 1$. Find the moment of inertia with respect to the z-axis if the mass density $\delta(x, y, z) = 1$ for all (x, y, z) in E.

Moment of inertia:

7. Find the circulation of the field $\mathbf{F}(x, y) = 3y\mathbf{i} + 2x\mathbf{j}$ along the ellipse C given by

$$\mathbf{r}(t) = \cos t\,\mathbf{i} + 3\sin t\,\mathbf{j}, \quad 0 \leq t \leq 2\pi.$$

Circulation:

8. Let E be the solid region bounded below by the plane $z = 0$, above by the sphere $x^2 + y^2 + z^2 = 9$, and on the sides by the cylinder $x^2 + y^2 = 4$.

a) Set up the integral that gives the volume of E using cylindrical coordinates and the order of integration $dz\,dr\,d\theta$. Do not evaluate the integral.

Answer:

b) Set up the integral that gives the volume of E using cylindrical coordinates and the order of integration $dr\,dz\,d\theta$. Do not evaluate the integral.

Answer:

9. Determine whether the following vector field is conservative in the plane. If it is conservative, find a potential function for $\mathbf{F}$.

$$\mathbf{F}(x, y) = \frac{2xy}{1+x^2}\,\mathbf{i} + \ln\left(1+x^2\right)\,\mathbf{j}.$$

Answer:

10. Evaluate the line integral

$$\int_C e^{x^2}\,dx + \left(x^3 - \ln(\sin y + 3)\right)\,dy,$$

where C is the counterclockwise oriented circle $x^2 + y^2 = 4$. Use whatever method you prefer.

Value: