

Math 231 Take Home Test 2

Due May 28, 2015

Name: _____

Info and Instructions:

1. Complete all problems in the space provided. In this exam, each problem is worth 10 points, for a total of 200 points.
2. Show ALL your work to receive full credit. You will get 0 credit for simply writing down the answers (unless it's a case of fill in the blank or state a definition, give an example, etc.)
3. Write neatly and logically so that I am able to follow your sequence of steps, box or otherwise indicate your answers where necessary.
4. No outside sources, you should be able to do all problems based on your class notes. I'm trusting that you will abide by this rule :p
5. Use the correct notation and write what you mean! x^2 and $x2$ are not the same thing, for example, and I will grade accordingly.
6. Use the conventions we've used in class to express your answers. Being able to explain your reasoning in a clear way is a very important part of this class. Any one of your classmates should be able to read your solution and understand it. (Note: solutions should NOT be shared before the test is collected, this is just an example of the level of detail I want you to use. Do not skip steps that someone with only a background of calculus 1 would miss.)
7. Under no circumstances will this test be accepted late. It is due first thing the morning of May 28, 2015. Be early and put your test on the desk.
8. I require physical copies of your completed test. Sending me scanned or electronic copies is not recommended. However, if you must send such, be sure that I receive it before 10:30am on May 14, 2015. And be sure that the test is scanned into a pdf document that will be legible when printed. No other formats are acceptable. I should be able to just open your file and press print with no trouble, and read the printout with no trouble. In any case, your solutions should be hand-written.
9. Your name should be on your test and it should be stapled. I will deduct 5 points otherwise.
10. Other than that, have fun and good luck!

1. Prove the principle of mathematical induction. That is, prove that:

For each positive integer n , let $P(n)$ be a statement. If

(1) $P(1)$ is true and

(2) The implication, if $P(k)$ then $P(k + 1)$ is true for every positive integer k ,

Then $P(n)$ is true for every positive integer n .

(Hint: a proof by contradiction is probably the easiest way. The Well-Ordering Principle would be used to obtain the contradiction.)

2. Use induction to prove that $1 + 5 + 9 + \cdots + (4n - 3) = 2n^2 - n$ for every positive integer n .

3. Prove that $n! > 2^n$ for every integer $n \geq 4$.

4. (a) Use mathematical induction to prove that every finite nonempty set of real numbers has a largest element.
(b) Use (a) to prove that every finite nonempty set of real numbers has a smallest element.

5. A sequence $\{a_n\}$ is defined recursively by $a_1 = 1, a_2 = 4, a_3 = 9$, and $a_n = a_{n-1} - a_{n-2} + a_{n-3} + 2(2n - 3)$ for $n \geq 4$. Conjecture a formula for a_n and prove that your conjecture is correct. Note, the *strong form* of mathematical induction may be needed.

6. How many positive integers between 1000 and 9999 inclusive are (a) divisible by 9? (b) divisible by 5 and 7? (c) divisible by 5 or 7? (d) divisible by 5 but not 7?

10. Assume everyone in California has three initials. Show that there are at least six people in California, population 37 million, with the same three initials who were born on the same day of the year (but not necessarily in the same year).

11. Show that if there are 100,000,000 wage earners in the U.S. who earn less than \$1,000,000 (but at least a penny), then there are two people who earned exactly the same amount of money, to the penny, last year.

12. A coin is flipped 10 times. How many possible outcomes (a) are there in total? (b) contain exactly three heads? (c) contain at most three heads? (d) contain the same number of heads and tails?

13. How many bit strings of length 10 have (a) exactly three 0s? (b) more 0s than 1s? (c) at least seven 1s? (d) at most three 1s?

14. Seven women and nine men are on the faculty in the mathematics department at a certain college. A committee of five members are to be selected, containing at least one man and at least one woman.

(a) How many such committees are possible?

(b) Suppose there is a love triangle of two men and one woman on the faculty. How many committees can be made that do NOT contain all three members of the love triangle?

15. Give a formula for the coefficient of x^k in the expansion of $\left(x + \frac{1}{x}\right)^{100}$, where k is an integer.

16. Show that a nonempty set of size n has the same number of subsets with an odd number of elements as it does with an even number of elements.

17. How many different bit strings can be formed using six 1s and eight 0s?

18. How many strings of seven or more characters can be formed from the letters in EVERGREEN?

19. How many integer solutions are there to the equation $x_1 + x_2 + x_3 + x_4 + x_5 = 21$ such that
(a) $x_1 \geq 2$? (b) $x_i \geq 3$ for $i = 1, 2, 3, 4, 5$? (c) $0 \leq x_1 < 10$? (d) $0 \leq x_1 \leq 3, 1 \leq x_2 < 4$?

20. How many different combinations of pennies, nickels, dimes, quarters and half dollars can a piggy bank contain if it has 20 coins?

Bonus: Due the day of the final!

I will add up to 10% to your exam grade if you do the following:

Write a paper, at least 6 pages in length, including a cover page, introduction, and references. In this paper, you are to discuss the *time complexity of algorithms*. And you must include examples to illustrate whatever you discuss. You may use any programming language or pseudo-programming language you wish to illustrate this, and do calculations to establish your examples. Make sure your paper addresses what the notion of time complexity means for algorithms; why should we care; common notations, conventions, and jargon for talking about time complexity; and how to calculate time complexity. It would also be good if you discuss some of the ways modern computer scientists try to cut down on the complexity of their algorithms.

Even if you don't need the extra points on your test grade, it would still be good for you to look into this. It will come in handy for your discrete data structures class.