THE CITY COLLEGE OF NEW YORK
DEPARTMENT OF MATHEMATICS
FALL 2016

MATH 392, FINAL EXAMINATION

YOUR NAME (print and sign):

NAME OF YOUR INSTRUCTOR:

Pr. 1	Pr. 2	Pr. 3	Pr. 4	Pr. 5	Pr. 6	Pr. 7	Pr. 8	Pr. 9	Pr. 10	Pr. 11	Total

INSTRUCTIONS:

- **There are a total of 11 problems.**
- **DO ALL PROBLEMS 1 THROUGH 7 AND THREE OF THE FOUR PROBLEMS 8-11. IN THE TABLE ABOVE, CROSS OUT ONE PROBLEM AMONG PROBLEMS 8-11 THAT YOU OMITTED.**
- **Each problem is worth 10 points.**
- **Notes, books and calculators are not to be used.**
- **All work on this exam is to be your own.**
- **Read each problem carefully. Be sure to show your work. Remember that it is your obligation to answer each question clearly and in a way that convinces the grader that you understand how to solve the problem.**
- **Stop working immediately at the end of the exam when time is called.**

1. Determine whether the vector field $\vec{F}$ given by

$$\vec{F}(x, y, z) = \frac{y}{x^2 + y^2}\vec{i} - \frac{x}{x^2 + y^2}\vec{j} + z^2\vec{k}$$

is conservative. Justify your claim.

2. Determine whether the following system of linear equations is consistent, and, if it is, find the set of solutions.

$$\begin{cases} 2x_2 + x_3 - x_4 = 1, \\ 2x_1 + 4x_2 + x_3 + x_4 = 3, \\ x_1 + x_2 + x_4 = 1, \\ x_1 + 5x_2 + 2x_3 - x_4 = 3. \end{cases}$$

What is the rank of the matrix of coefficients and how many (if any) free variables does the system have?

3. Find the work done by the vector field

$$\vec{F}(x,y,z) = 2y\vec{i} + e^x\vec{j} - z^3\vec{k}$$

moving a particle along the curve C given by

$$\vec{r}(t) = (t^2, t^2+2, t), \quad 0 \le t \le 1.$$

4. Use Green's Theorem to find the line integral

$$\int_C \vec{F} \cdot d\vec{r},$$

where $\vec{F}(x, y) = -y\vec{i} + x^2\vec{j}$, and

$$C = \{(x, y)\colon x^2 + |y - 1| = 2\},$$

oriented counterclockwise.

5. Let $\vec{F}(x,y,z) = 2x\vec{i} - y\vec{k}$ and S be the boundary surface of a region E in $\mathbb{R}^3$ given by

$$E = \{(x,y,z)\colon z \geq x^2 + y^2,\ x + y + \frac{1}{2}z \leq 1\}.$$

We assume that S is oriented outward with respect to E. Use the Divergence Theorem to compute

$$\iint_S \vec{F} \cdot d\vec{S}.$$

6. Approximate to three decimal digits the surface area of the patch on the graph of

$$z = -x^5 - y^2 + e^{xy-1},$$

for $1 \leq x \leq 1.1,\ 1 \leq y \leq 1.2$. You may assume that $\sqrt{2} \approx 1.4$.

7. Solve the following linear system of ODE's.

$$\begin{cases} y_1' = 2(y_1 + y_2), \\ y_2' = -y_1 - 2y_2. \end{cases}$$

8. Find the arc-length parameter s (as a function of t) of the curve given by

$$\vec{r}(t) = (e^t, e^{-t}, \sqrt{2}\, t), \quad 0 \leq t \leq 1.$$

What is the length of this curve?

9. Let $\vec{F}(x,y,z) = y\vec{\imath} + (5-z)\vec{\jmath} + 2x\vec{k}$, and let C be the circle in the plane

$$x + y + 2z = 1,$$

centered at the point $(2,-1,0)$ and whose radius is 2. Assume that C is oriented counter-clockwise when viewed from the origin. Find the line integral

$$\int_C \vec{F} \cdot d\vec{r}.$$

10. Let

$$\vec{F}(x,y,z) = x^2\vec{i} - \vec{j} - 2yz\vec{k}.$$

Is this vector field the curl of some vector field $\vec{G}$ in some open region U in $\mathbb{R}^3$? Justify your claim.

11. Let

$$A = \begin{bmatrix} 0 & 2 & 1 & 1 \\ -1 & 1 & 0 & 2 \\ 3 & 3 & 1 & -1 \\ 0 & 0 & 2 & 1 \end{bmatrix}.$$

Find $\det\left(2AA^T(A^{-1})^3\right)$.