11.5 – Solve Systems of Nonlinear Equations

Learning Objectives

1. Solve a System of Nonlinear Equations Using Substitution
2. Solve a System of Nonlinear Equations Using Elimination

Comments

A **system of nonlinear equations** is a system where at least one of the equations is not linear.

In solving systems of nonlinear equations, the solution(s) also represent(s) the point(s) of intersection (if any) of the graphs of the equations.

There is no general method for solving a system of nonlinear equations. Sometimes substitution is best; other times elimination is best; and sometimes neither of these methods works. Experience and a certain degree of imagination are your allies here.

1. If the system contains two variables and if the equations in the system are easy to graph, then graph them. By graphing each equation in the system, you can get an idea of how many solutions a system has and approximate their location.
2. Extraneous solutions can creep in when solving nonlinear systems, so it is imperative to check all apparent solutions.

1 – Solve a System of Nonlinear Equations Using Substitution

The substitution method is an algebraic method that will work well in many situations. It works especially well when it is easy to solve one of the equations for one of the variables. The substitution method is very similar to the substitution method that we used for systems of linear equations.

HOW TO

Solve a system of nonlinear equations by substitution.

Step 1. Identify the graph of each equation. Sketch the possible options for intersection.
Step 2. Solve one of the equations for either variable.
Step 3. Substitute the expression from Step 2 into the other equation.
Step 4. Solve the resulting equation.
Step 5. Substitute each solution in Step 4 into one of the original equations to find the other variable.
Step 6. Write each solution as an ordered pair.
Step 7. Check that each ordered pair is a solution to **both** original equations.

Example 1 – Solving a System of Nonlinear Equations Using Substitution

Solve the following system of equations:

$$\begin{cases} 3x - y = -2 \\ 2x^2 - y = 0 \end{cases}$$

Solution:

Example 2 – Solving a System of Nonlinear Equations Using Substitution

Solve the system by using substitution: $\begin{cases} 9x^2 + y^2 = 9 \\ y = 3x - 3 \end{cases}$.

Solution:

Example 3 – Solving a System of Nonlinear Equations Using Substitution

Solve the system by using substitution: $\begin{cases} x^2 - y = 0 \\ y = x - 2 \end{cases}$.

Solution:

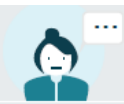
Example 4 – Solving a System of Nonlinear Equations Using Substitution

Solve the system by using substitution: $\begin{cases} x^2 - y = 0 \\ y = x - 2 \end{cases}$.

Solution:

2 – Solve a System of Nonlinear Equations Using Elimination

When we studied systems of linear equations, we used the method of elimination to solve the system. We can also use elimination to solve systems of nonlinear equations. It works well when the equations have both variables squared. When using elimination, we try to make the coefficients of one variable to be opposites, so when we add the equations together, that variable is eliminated.



HOW TO

Solve a system of equations by elimination.

Step 1. Identify the graph of each equation. Sketch the possible options for intersection.
Step 2. Write both equations in standard form.
Step 3. Make the coefficients of one variable opposites.
Decide which variable you will eliminate.
Multiply one or both equations so that the coefficients of that variable are opposites.
Step 4. Add the equations resulting from Step 3 to eliminate one variable.
Step 5. Solve for the remaining variable.
Step 6. Substitute each solution from Step 5 into one of the original equations. Then solve for the other variable.
Step 7. Write each solution as an ordered pair.
Step 8. Check that each ordered pair is a solution to **both** original equations.

Example 5 – Solving a System of Nonlinear Equations Using Elimination

$$\text{Solve: } \begin{cases} x^2 + y^2 = 13 \\ x^2 - y = 7 \end{cases}$$

Solution:

Example 6 – Solving a System of Nonlinear Equations Using Elimination

Solve the system by elimination: $\begin{cases} x^2 + y^2 = 4 \\ x^2 - y = 4 \end{cases}$.

Solution:

Example 7 – Solving a System of Nonlinear Equations Using Elimination

Find all solutions of the system.

$$\begin{cases} 3x^2 + 2y = 26 \\ 5x^2 + 7y = 3 \end{cases}$$

Solution: