

11.3 – Ellipses

Learning Objectives

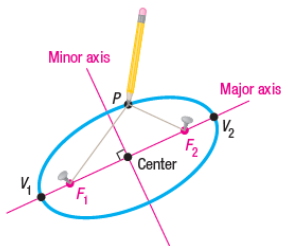
- 1 Graph an ellipse with center at the origin
- 2 Find the equation of an ellipse with center at the origin
- 3 Graph an ellipse with center not at the origin

1 – Graph an Ellipse with Center at the Origin

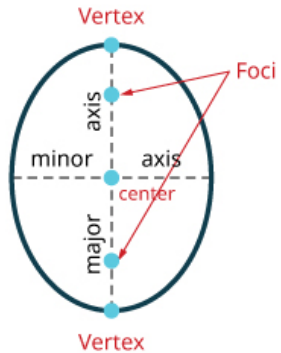
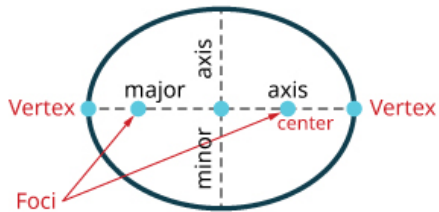
Definition – Ellipse

An **ellipse** is all points in a plane where the sum of the distances from two fixed points is constant. Each of the fixed points is called a **focus** of the ellipse.

Characteristics of an Ellipse



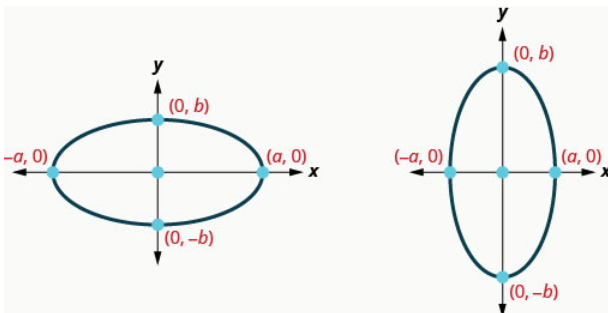
In figure above, the foci are labeled F_1 and F_2 . The line containing the foci is called the major axis. The midpoint of the line segment joining the foci is the center of the ellipse. The line through the center and perpendicular to the major axis is the minor axis. The two points of intersection of the ellipse and the major axis are the vertices, V_1 and V_2 , of the ellipse. The distance from one vertex to the other is the length of the major axis. The ellipse is symmetric with respect to its major axis, with respect to its minor axis, and with respect to its center.



Standard Form of the Equation an Ellipse with Center $(0, 0)$

The standard form of the equation of an ellipse with center $(0, 0)$, is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



Ellipse with Center $(0, 0)$		
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$a > b$	$b > a$
Major axis	on the x - axis.	on the y -axis.
x-intercepts	$(-a, 0), (a, 0)$	
y-intercepts	$(0, -b), (0, b)$	

How to Graph an Ellipse with Center $(0, 0)$.

- Step 1. Write the equation in standard form.
- Step 2. Determine whether the major axis is horizontal or vertical.
- Step 3. Find the endpoints of the major axis.
- Step 4. Find the endpoints of the minor axis
- Step 5. Sketch the ellipse.

Example 1 – Graphing an Ellipse with center $(0, 0)$

Graph: $\frac{x^2}{4} + \frac{y^2}{9} = 1.$

Solution:

Example 2 – Graphing an Ellipse with center $(0, 0)$

Graph: $x^2 + 4y^2 = 16$.

Solution:

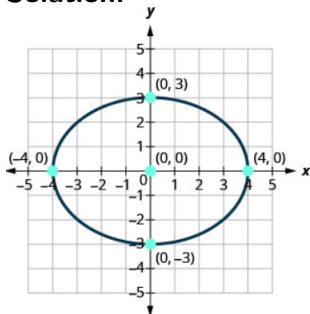
2 – Find the Equation of an Ellipse with Center at the Origin

If we are given the graph of an ellipse, we can find the equation of the ellipse.

Example 3

Find the equation of the ellipse shown below.

Solution:



3 – Graph an Ellipse with Center Not at the Origin

The ellipses we have looked at so far have all been centered at the origin. We will now look at ellipses whose center is (h, k) .

Standard Form of the Equation an Ellipse with Center (h, k)

The standard form of the equation of an ellipse with center (h, k) , is

$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$$

When $a > b$, the major axis is horizontal so the distance from the center to the vertex is a .

When $b > a$, the major axis is vertical so the distance from the center to the vertex is b .

Example 4

Graph: $\frac{(x-3)^2}{9} + \frac{(y-1)^2}{4} = 1$

Solution:

Example 5 – Analyzing the Equation of an Ellipse

Write the equation $x^2 + 4y^2 - 4x + 24y + 24 = 0$ in standard form and sketch its graph.

Solution: