11.1 Distance and Midpoint Formulas; Circles

Learning Objectives

1. Use the Distance Formula
2. Use the Midpoint Formula
3. Write the equation of a circle in standard form
4. Work with the general form of the equation of a circle

Theorem – Distance Formula
The distance between two points $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$, denoted by $d(P_1, P_2)$, is

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example 1 – Using the Distance Formula

Use the Distance Formula to find the distance between the points $(-5, -3)$ and $(7, 2)$.

Solution:

Example 2 – Using the Distance Formula

Use the Distance Formula to find the distance between the points $(10, -4)$ and $(-1, 5)$.

Solution:

2 – Use the Midpoint Formula

It is often useful to be able to find the midpoint of a segment. For example, if you have the endpoints of the diameter of a circle, you may want to find the center of the circle which is the midpoint of the diameter. To find the midpoint of a line segment, we find the average of the $x-$coordinates and the average of the y-coordinates of the endpoints.

Theorem – Midpoint Formula
The midpoint $M = (x, y)$ of a line segment from $P_1 = (x_1, y_1)$ to $P_2 = (x_2, y_2)$ is

$$M = (x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Example 3 – Finding the Midpoint of a Line Segment

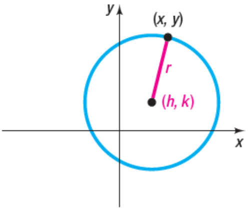
Find the midpoint of the line segment from $P_1 = (-5, 5)$ to $P_2 = (3, 1)$. Plot the points P_1 and P_2 and their midpoint.

Solution:

3 – Write the Equation of a Circle in Standard Form

Definition – Circle

A **circle** is a set of points in the $xy-$plane that are a fixed distance r from a fixed point (h, k). The fixed distance r is called the radius, and the fixed point (h, k) is called the center of the circle.



Definition – Standard Form of an Equation of a Circle

The standard form of an equation of a circle with radius r and center (h, k) is $(x-h)^2+(y-k)^2=r^2$. If the center of the circle is $(0, 0)$, then the equation is $x^2+y^2=r^2$.

Example 4 – Writing the Standard Form of the Equation of a Circle

Write the standard form of the equation of the circle with radius 5 and center $(-3, 6)$.

Solution:

Example 5 – Writing the Standard Form of the Equation of a Circle

Write the standard form of the equation of the circle with center $(2, 4)$ that also contains the point $(-2, 1)$.
Solution:

Example 6 – Graphing a Circle

Graph the equation: $(x + 3)^2 + (y - 2)^2 = 16$

Solution:

Example 7 – Finding the Intercepts of a Circle

Find the intercepts, if any, of the graph of the circle $(x+3)^2+(y-2)^2=16$.

Solution:

4 – Work with the General Form of the Equation of a Circle

If we eliminate the parentheses from the standard form of the equation of the circle given in examples 7 and 8, we get

$$(x+3)^2+(y-2)^2=16$$

$$x^2+6x+9+y^2-4y^2+4=16$$

which simplifies to

$$x^2+y^2+6x-4y-3=0$$

It can be shown that any equation of the form

$$x^2+y^2+ax+by+c=0$$

has a graph that is a circle, is a point, or has no graph at all.

Definition – General Form of the Equation of a Circle
When its graph is a circle, the equation

$$x^2 + y^2 + ax + by + c = 0$$

is the **general form of the equation of a circle**.

If an equation of a circle is in general form, we use the method of completing the square to put the equation in standard form so that we can identify its center and radius.

Example 8 – Review of Completing the Square

Complete the square to make a perfect square trinomial. Then write the result as a binomial squared.

$$x^2 - 26x$$

Solution:

Example 9 – Identifying an Equation of a Circle

Show that the equation $x^2 + y^2 + 2x - 6y + 7 = 0$ represents a circle, and find the center and radius of the circle.

Solution: