9.5 - Solving Trigonometric Equations

Learning Objectives

1. Solve equations involving a single trigonometric function.
2. Solve trigonometric equations using inverse trig functions and a calculator.
3. Solve trigonometric equations that are quadratic in form.

1 – Solving Equations Involving a Single Trigonometric Function

Trigonometric Equations are equations involving trigonometric functions of unknown angles. There are two types of trigonometric equations.

1. Trigonometric Identities – if they are satisfied by all values of the unknown angles for which the functions are defined. We discussed trig identities in section 9.1.

2. Conditional equations, or simply equations, if they are satisfied only by a particular value of the unknown angles. In this section we focus our attention on solving conditional trig equations.

To solve a trigonometric equation, we find all the values of the variable that make the equation true.

Example 1 – Checking Whether a Given Number Is a Solution of a Trigonometric Equation

Determine whether $\theta = \dfrac{\pi}{4}$ is a solution of the equation $2\sin\theta - 1 = 0$. Is $\theta = \dfrac{\pi}{6}$ a solution?

Solution:

Solving Trigonometric Equations

To solve trigonometric equations,we often start by isolating the trig function, just like we would in algebra. Then we find solutions within a given interval or all possible solutions.

Because trig functions are periodic, their values repeat. For example, sine and cosine repeat every 2π. That means if θ is a solution, so is $\theta \pm 2\pi k$, where k is any integer.

This pattern applies to other trig functions too. Solving these equations also uses familiar techniques: factoring, simplifying, using identities, and checking the domain. Trig identities can make the process easier and help us find all valid solutions.

Note: Unless the domain of the variable is restricted to an interval, we need to find all the solutions of a trigonometric equation. To do this we first find solutions over an interval whose length equals the period of the function and then add multiples of that period to the solutions found.

Example 2 – Finding All the Solutions of a Trigonometric Equation

Solve the equation: $\cos\theta = \dfrac{1}{2}$

Give a general formula for all the solutions. List eight of the solutions.

Solution:

Example 3 – Solving a Linear Trigonometric Equation

Solve the equation: $2 \sin \theta + \sqrt{3} = 0, \quad 0 \leq \theta < 2\pi$

Solution:

Example 4 – Solving a Problem Involving a Single Trigonometric Function

Solve the problem exactly: $2\sin^2\theta - 1 = 0, 0 \leq \theta < 2\pi$.

Solution:

Example 5 – Solving an Equation Involving Tangent

Solve the equation: $\tan\left(\theta - \frac{\pi}{2}\right) = 1, \quad 0 \leq \theta < 2\pi$

Solution:

Example 6 – Solving an Equation Involving Secant

Solve the equation: $\sec^2\theta - 2 = 0$, for all values of θ on the interval $[0, 2\pi]$.

Solution:

2 – Solve Trigonometric Equations Using a Calculator

Not all trigonometric equations can be solved exactly using the unit circle, especially when the angle is not one of the special angles. In these cases, we use a calculator and inverse trigonometric functions to find approximate solutions.

Example 7 – Solving Trigonometric Equations Using a Calculator and Inverse Functions

Solve the equations:

(a) $\sin\theta = \dfrac{1}{2}$ **(b)** $\cos\theta = 0.4$

Give a general formula for all the solutions and list all the solutions in the interval $[-2\pi, 2\pi]$.

Solution:

Many trigonometric equations can be solved using techniques that we already know, such as using the quadratic formula (if the equation is a second-degree polynomial) or factoring. Factoring is one of the most useful techniques for solving equations, including trigonometric equations. The idea is to move all terms to one side of the equation, factor, and then use the Zero-Product Property.

Example 8 – Solving a Trigonometric Equation Quadratic in Form

Solve the equation: $2\sin^2\theta - 3\sin\theta + 1 = 0, \quad 0 \leq \theta < 2\pi$

Solution:

Example 9 – Solving a Trigonometric Equation by Factoring

Solve the equation $5 \sin \theta \cos \theta + 4 \cos \theta = 0$.

Solution:

Example 10 – Solving a Trigonometric Equation by using the Quadratic Formula

Solve the equation exactly: $\cos^2\theta + 3\cos\theta - 1 = 0, 0 \leq \theta < 2\pi.$

Solution: