9.3 – Double-angle and Half-angle Formulas

Learning Objectives

1. Use Double-Angle Formulas to Find Exact Values
2. Use Half-Angle Formulas to Find Exact Values

1 – Use Double-Angle Formulas to Find Exact Values

Theorem – Double-Angle Formulas for Sine and Cosine

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$

$$\cos(2\theta) = 1 - 2\sin^2\theta$$

$$\cos(2\theta) = 2\cos^2\theta - 1$$

The Double-Angle Formulas allow us to find the values of sine and cosine at $2x$ from their values at x. The formulas are immediate consequences of the Sum Formulas.

Example 1 – Use Double-Angle Formulas to Find Exact Values

If $\sin\theta = \frac{3}{5}$, $\frac{\pi}{2} < \theta < \pi$, find the exact value of:

(a) $\sin(2\theta)$ (b) $\cos(2\theta)$

Solution:

Example 2 – Use Double-Angle Formulas to Find Exact Values

If $\cos x = -\frac{2}{3}$ and x is in Quadrant II, find $\cos 2x$ and $\sin 2x$.

Solution:

2 – Use Half-Angle Formulas to Find Exact Values

Theorem – Half-Angle Formulas for Sine and Cosine

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$$

$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$$

where $+$ or $-$ sign is determined by the quadrant of the angle $\frac{\alpha}{2}$.

The Half-Angle Formulas relate the values of sine and cosine at $\frac{1}{2}x$ to their values at x.

Example 3 – Finding Exact Values Using Half-angle Formulas

Use a Half-angle Formula to find the exact value of:

(a) $\cos 15°$ (b) $\sin(-15°)$

Solution:

Example 4 – Finding Exact Values Using Half-angle Formulas

If $\cos\alpha = -\frac{3}{5}, \pi < \alpha < \frac{3\pi}{2}$, find the exact value of:

(a) $\sin\frac{\alpha}{2}$ (b) $\cos\frac{\alpha}{2}$

Solution:

Example 5 – Using a Half-Angle Formula

Find the exact value of $\sin 22.5°$.

Solution: