9.2 – Sum and Differences Identities

Learning Objectives

1. Use Sum and Difference Formulas to Find Exact Values
2. Use Sum and Difference Formulas to Establish Identities

1 – Use Sum and Difference Formulas to Find Exact Values

Finding the exact value of the sine or cosine of an angle is often easier if we can rewrite the given angle in terms of two angles that have known trigonometric values. We use the **Sum and Difference Formulas** to do this.

Sum and Difference Formulas

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

Example 1 – Using the Sum and Difference Formulas for Cosine

Find the exact value of each expression.

(a) $\cos 75°$ **(b)** $\cos \frac{\pi}{12}$

Solution:

Example 2 – Using the Sum Formula to Find an Exact Value

Find the exact value of $\sin \frac{7\pi}{12}$.

Solution:

Example 3 – Using the Difference Formula to Find an Exact Value

Find the exact value of $\sin 80° \cos 20° - \cos 80° \sin 20°$.

Solution:

Example 4 – Finding Exact Values

If $\sin\alpha = \frac{4}{5}$, $\frac{\pi}{2} < \alpha < \pi$, and $\sin\beta = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$, $\pi < \beta < \frac{3\pi}{2}$, find the exact value of each of the following.

(a) $\cos\alpha$ (b) $\cos\beta$ (c) $\cos(\alpha + \beta)$ (d) $\sin(\alpha + \beta)$

Solution:

2 – Use Sum and Difference Formulas to Establish Identities

We can use the Sum and Difference Formulas to establish the Cofunction Identities (from section 7.2).

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$$

Example 5 – Establishing a Cofunction Identity

Establish the cofunction identity $\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$.

Solution:

Example 6 – Establishing an Identity

Establish the identity: $\dfrac{\cos(\alpha - \beta)}{\sin\alpha \sin\beta} = \cot\alpha \cot\beta + 1$

Solution:

Example 7 – An Identity from Calculus

If $f(x) = \sin x$, show that

$$\frac{f(x+h) - f(x)}{h} = \sin x \left(\frac{\cos h - 1}{h} \right) + \cos x \left(\frac{\sin h}{h} \right)$$

Solution: