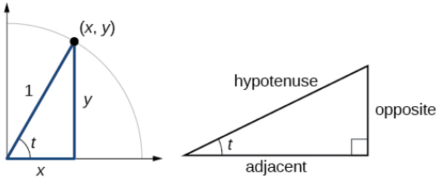
# 7.2 – Right Triangle Trigonometry

Learning Objectives

1. Use right triangles to evaluate trigonometric functions.
2. Find function values for $30^\circ \left(\frac{\pi}{6}\right)$, $45^\circ \left(\frac{\pi}{4}\right)$, and $60^\circ \left(\frac{\pi}{3}\right)$
3. Use equal cofunctions of complementary angles.
4. Use the definitions of trigonometric functions of any angle.
5. Use right-triangle trigonometry to solve applied problems.

# 1 – Using Right Triangles to Evaluate Trigonometric Functions

The picture shows a right triangle with a vertical side of length $y$ and a horizontal side has length $x$. Notice that the triangle is inscribed in a circle of radius 1. Such a circle, with a center at the origin and a radius of 1, is known as a **unit circle**.



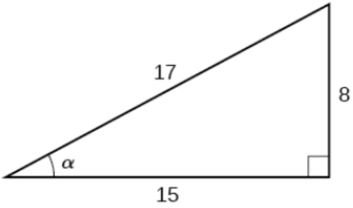

We can define the trigonometric functions in terms an angle $t$ and the lengths of the sides of the triangle. The **adjacent side** is the side closest to the angle, $x$. The **opposite side** is the side across from the angle, $y$. The **hypotenuse** is the side of the triangle opposite the right angle, 1.

Given a right triangle with an acute angle of the first three trigonometric functions are listed.

$$\begin{aligned} \text{Sine} \qquad \sin t &= \frac{\text{opposite}}{\text{hypotenuse}} \\ \text{Cosine} \qquad \cos t &= \frac{\text{adjacent}}{\text{hypotenuse}} \\ \text{Tangent} \qquad \tan t &= \frac{\text{opposite}}{\text{adjacent}} \end{aligned}$$

# Example 1 – Evaluating a Trigonometric Function of a Right Triangle

Given the triangle shown below, find the value of $\cos \alpha$.




**Solution**:

# Reciprocal Functions

In addition to sine, cosine, and tangent, there are three more functions. These too are defined in terms of the sides of the triangle.

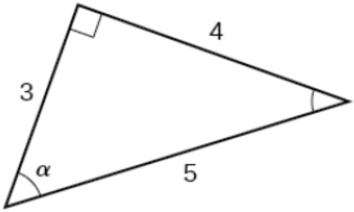
$$\begin{array}{lll} \text{Secant} & \sec t & = \dfrac{\text{hypotenuse}}{\text{adjacent}} \\ \text{Cosecant} & \csc t & = \dfrac{\text{hypotenuse}}{\text{opposite}} \\ \text{Cotangent} & \cot t & = \dfrac{\text{adjacent}}{\text{opposite}} \end{array}$$

Take another look at these definitions. These functions are the reciprocals of the first three functions.

$$\begin{array}{lllllll} \sin t & = & \dfrac{1}{\csc t} & \qquad & \csc t & = & \dfrac{1}{\sin t} \\ \cos t & = & \dfrac{1}{\sec t} & & \sec t & = & \dfrac{1}{\cos t} \\ \tan t & = & \dfrac{1}{\cot t} & & \cot t & = & \dfrac{1}{\tan t} \end{array}$$

# Example 2 – Evaluating Trigonometric Functions of Angles

Using the triangle shown below, evaluate $\sin\alpha$, $\cos\alpha$, $\tan\alpha$, $\sec\alpha$, $\csc\alpha$, and $\cot\alpha$.

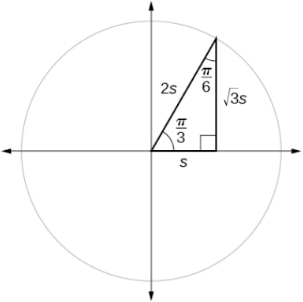


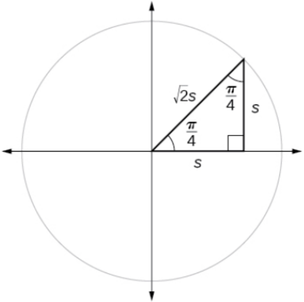

**Solution**:

# 2 – Finding Trigonometric Functions of Special Angles Using Side Lengths

Suppose we have a $30^\circ$, $60^\circ$, $90^\circ$ triangle, which can also be described as a $\frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}$ triangle. The sides have lengths in the relation $s$, $\sqrt{3}s$, $2s$.

The sides of a $45^\circ$, $45^\circ$, $90^\circ$ triangle, which can also be described as a $\frac{\pi}{4}, \frac{\pi}{4}, \frac{\pi}{2}$ triangle. The sides have lengths in the relation $s$, $s$, $\sqrt{2}s$.



# Example 3 – Evaluating Trigonometric Functions of Special Angles Using Side Lengths

Find the exact value of the trigonometric functions of $\frac{\pi}{3}$, using side lengths.

**Solution**:

# 3 – Using Equal Cofunction of Complements

**Cofunction Identities**

The cofunction identities in radians are

| | |
|---|---|
| $\cos\ t = \sin\left(\frac{\pi}{2} - t\right)$ | $\sin\ t = \cos\left(\frac{\pi}{2} - t\right)$ |
| $\tan\ t = \cot\left(\frac{\pi}{2} - t\right)$ | $\cot\ t = \tan\left(\frac{\pi}{2} - t\right)$ |
| $\sec\ t = \csc\left(\frac{\pi}{2} - t\right)$ | $\csc\ t = \sec\left(\frac{\pi}{2} - t\right)$ |

**HOW TO**

**Given the sine and cosine of an angle, find the sine or cosine of its complement.**

1. To find the sine of the complementary angle, find the cosine of the original angle.
2. To find the cosine of the complementary angle, find the sine of the original angle.

# Example 4 – Using Cofunction Identities

If $\sin t = \dfrac{5}{12}$, find $\cos\left(\dfrac{\pi}{2} - t\right)$.

**Solution**:

# 4 – Using Trigonometric Functions

In previous examples, we evaluated the sine and cosine in triangles where we knew all three sides. But the real power of right-triangle trigonometry emerges when we look at triangles in which we know an angle but do not know all the sides.

A triangle has six parts: three angles and three sides. To **solve a triangle** means to determine all of its parts from the information known about the triangle, that is, to determine the lengths of the three sides and the measures of the three angles.

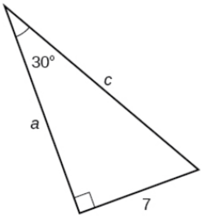
**HOW TO**

**Given a right triangle, the length of one side, and the measure of one acute angle, find the remaining sides.**

1. For each side, select the trigonometric function that has the unknown side as either the numerator or the denominator. The known side will in turn be the denominator or the numerator.
2. Write an equation setting the function value of the known angle equal to the ratio of the corresponding sides.
3. Using the value of the trigonometric function and the known side length, solve for the missing side length.

# Example 5 – Finding Missing Side Lengths Using Trigonometric Ratios

Find the unknown sides of the triangle shown below.



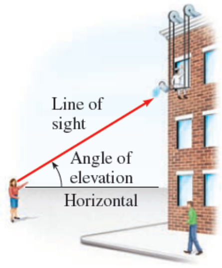

**Solution**:

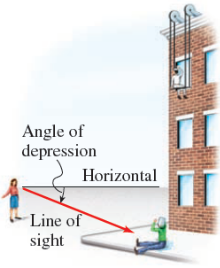
# 5 – Using Right Triangle Trigonometry to Solve Applied Problems

Right-triangle trigonometry has many practical applications. For example, the ability to compute the lengths of sides of a triangle makes it possible to find the height of a tall object without climbing to the top or having to extend a tape measure along its height.

The **angle of elevation** of an object above an observer relative to the observer is the angle between the horizontal and the line from the object to the observer's eye.

The **angle of depression** of an object below an observer relative to the observer is the angle between the horizontal and the line from the object to the observer's eye.




**HOW TO**

**Given a tall object, measure its height indirectly.**

1. Make a sketch of the problem situation to keep track of known and unknown information.
2. Lay out a measured distance from the base of the object to a point where the top of the object is clearly visible.
3. At the other end of the measured distance, look up to the top of the object. Measure the angle the line of sight makes with the horizontal.
4. Write an equation relating the unknown height, the measured distance, and the tangent of the angle of the line of sight.
5. Solve the equation for the unknown height.

# Example 6 – Measuring a Distance Indirectly

To find the height of a tree, a person walks to a point 30 feet from the base of the tree. She measures an angle of $57^\circ$ between a line of sight to the top of the tree and the ground, as shown below. Find the height of the tree.

**Solution**: