6.6 – Exponential and Logarithmic Equations

Learning Objectives

1. Use the method of common bases to solve exponential equations.
2. Use logarithms to solve exponential equations.
3. Use the definition of a logarithm to solve logarithmic equations.
4. Use the one-to-one property of logarithms to solve logarithmic equations.

Use the Method of Common Bases to Solve Exponential Equations

Method of Common Bases

If $b^x = b^y$ then $x = y$.

When two exponential expressions with the same base are equal, then their exponents are equal.

This technique can be used in any situation where all bases involved can be written with a common base. In a practical sense, this is rather rare. Yet, these types of algebraic manipulations help us see the **structure** in **exponential expressions**.

Example 1 – Solving an Exponential Equation with a Common Base

Solve $2^{x-1} = 2^{2x-4}$ for x.

Solution:

Example 2 – Solving Equations by Rewriting Them to Have a Common Base

Solve $8^{x+2} = 16^{x+1}$ for x.

Solution:

Example 3 – Solving Equations by Rewriting Roots with Fractional Exponents to Have a Common Base

Solve $2^{5x} = \sqrt{2}$ for x.

Solution:

Example 4 – Solving an Equation with Positive and Negative Powers

Solve $3^{x+1} = -2$ for x.

Solution:

2 – Solving Exponential Equations Using Logarithms

Sometimes the terms of an exponential equation cannot be rewritten with a common base. In these cases, we solve by taking the logarithm of each side.

HOW TO

Given an exponential equation in which a common base cannot be found, solve for the unknown.

1. Apply the logarithm of both sides of the equation.
   a. If one of the terms in the equation has base 10, use the common logarithm.
   b. If none of the terms in the equation has base 10, use the natural logarithm.
2. Use the rules of logarithms to solve for the unknown.

Example 5 – Solving an Equation Containing Powers of Different Bases

Solve $5^{x+2} = 4^x$ for x.

Solution:

Example 6 – Solving an Equation with base e

Solve each equation below.

(a) $100 = 20e^{2t}$ (b) $4e^{2x} + 5 = 12$

Solution:

3 – Using the Definition of a Logarithm to Solve Logarithmic Equations

Using the Definition of a Logarithm to Solve Logarithmic Equations

For any algebraic expression S and real numbers b and c, where $b > 0,\ \ b \neq 1,$

$$\log_b(S) = c \quad \text{if and only if} \quad b^c = S$$

Example 7 – Using Algebra to Solve a Logarithmic Equations

Solve each equation below.

(a) $2\ln x + 3 = 7$ (b) $2\ln(6x) = 7$

Solution:

Extraneous Solutions

Sometimes the methods used to solve an equation introduce an **extraneous solution**, which is a solution that is correct algebraically but does not satisfy the conditions of the original equation.

4 – Using the One-to-One Property of Logarithms to Solve Logarithmic Equations

As with exponential equations, we can use the one-to-one property to solve logarithmic equations.

USING THE ONE-TO-ONE PROPERTY OF LOGARITHMS TO SOLVE LOGARITHMIC EQUATIONS

For any algebraic expressions S and T and any positive real number b, where $b \neq 1$,

$$\log_b S = \log_b T \quad \text{if and only if} \quad S = T$$

Note, when solving an equation involving logarithms, always check to see if the answer is correct or if it is an extraneous solution.

Example 8 – Solving an Equation Using the One-to-One Property of Logarithms

Solve $\ln(x^2) = \ln(2x + 3)$ for x.

Solution: