5.4 – Dividing Polynomials

Learning Objectives

1. Use long division to divide polynomials.
2. Use synthetic division to divide polynomials.

1 – Using Long Division to Divide Polynomials

The Division Algorithm

The Division Algorithm states that, given a polynomial dividend $P(x)$ and a non-zero polynomial divisor $D(x)$ where the degree of $D(x)$ is less than or equal to the degree of $P(x)$, there exist unique polynomials $Q(x)$ and $R(x)$ such that

$$\frac{P(x)}{D(x)} = Q(x) + \frac{R(x)}{D(x)} \qquad \text{or} \qquad P(x) = D(x) \cdot Q(x) + R(x)$$

Dividend, Divisor, Quotient, Remainder

The polynomials $P(x)$ and $D(x)$ are called the **dividend** and **divisor**, respectively, $Q(x)$ is the **quotient**, and $R(x)$ is the **remainder**.

HOW TO

Given a polynomial and a binomial, use long division to divide the polynomial by the binomial.

1. Set up the division problem.
2. Determine the first term of the quotient by dividing the leading term of the dividend by the leading term of the divisor.
3. Multiply the answer by the divisor and write it below the like terms of the dividend.
4. Subtract the bottom binomial from the top binomial.
5. Bring down the next term of the dividend.
6. Repeat steps 2–5 until reaching the last term of the dividend.
7. If the remainder is non-zero, express as a fraction using the divisor as the denominator.

Example 1 – Using Long Division to Divide a Second-Degree Polynomial

Divide $5x^2 + 3x - 2$ by $x + 1$.
Solution:

Example 2 – Using Long Division to Divide a Third-Degree Polynomial

Divide $6x^3 + 11x^2 - 31x + 15$ by $3x - 2$.

Solution:

Look back at the dividends in previous examples. The terms were written in descending order of degrees, and there were no missing degrees. The dividend in the next example will be $x^4 - x^2 + 5x - 6$. It is missing an x^3 term. We will add in $0x^3$ as a placeholder.

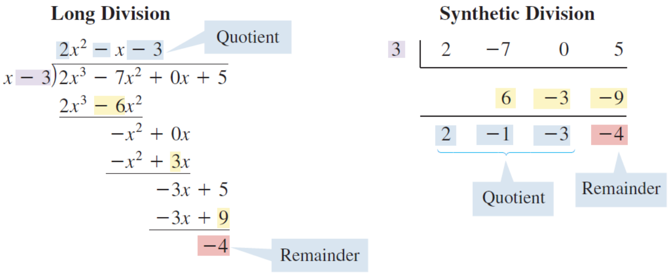
Example 3 – Using Long Division to Divide a Fourth-Degree Polynomial

Find the quotient (Divide): $(x^4 - x^2 + 5x - 6) \div (x + 2)$.
Solution:

2 – Using Synthetic Division to Divide Polynomials

As we've seen, long division of polynomials can involve many steps and be quite cumbersome.

Synthetic division is a shorthand method of dividing polynomials for the special case of dividing by a linear factor whose leading coefficient is 1.



Synthetic Division

Synthetic division is a shortcut that can be used when the divisor is a binomial in the form $x - k$ where $k$is a real number. In synthetic division, only the coefficients are used in the division process.

HOW TO

Given two polynomials, use synthetic division to divide.

1. Write k for the divisor.
2. Write the coefficients of the dividend.
3. Bring the lead coefficient down.
4. Multiply the lead coefficient by k. Write the product in the next column.
5. Add the terms of the second column.
6. Multiply the result by k. Write the product in the next column.
7. Repeat steps 5 and 6 for the remaining columns.
8. Use the bottom numbers to write the quotient. The number in the last column is the remainder and has degree 0, the next number from the right has degree 1, the next number from the right has degree 2, and so on.

Example 4 – Using Synthetic Division to Divide a Second-Degree Polynomial

Use synthetic division to divide $5x^2 - 3x - 36$ by $x - 3$.

Solution:

Example 5 – Using Synthetic Division to Divide a Third-Degree Polynomial

Use synthetic division to divide $4x^3 + 10x^2 - 6x - 20$ by $x + 2$.
Solution:

Example 6 – Using Synthetic Division to Divide a Fourth-Degree Polynomial

Use synthetic division to divide $-9x^4 + 10x^3 + 7x^2 - 6$ by $x - 1$.
Solution: