5.1 – Quadratic Functions

Learning Objectives

1. Recognize the graph of a quadratic function
2. Find the axis of symmetry and vertex of a parabola
3. Find the intercepts of a parabola
4. Graph quadratic functions using properties
5. Solve maximum and minimum applications

1 – Recognize the Graph of a Quadratic Function

Forms of Quadratic Functions

A quadratic function is a polynomial function of degree two. The graph of a quadratic function is a parabola.

The **general form of a quadratic function** is $f(x) = ax^2 + bx + c$ where a, b, and c are real numbers and $a \neq 0$.

The **standard form of a quadratic function** is $f(x) = a(x - h)^2 + k$ where $a \neq 0$.

The vertex (h, k) is located at

$$h = -\frac{b}{2a}, \quad k = f(h) = f\left(\frac{-b}{2a}\right)$$

Domain and Range of a Quadratic Function

The domain of any quadratic function is all real numbers unless the context of the function presents some restrictions.

The range of a quadratic function written in general form $f(x) = ax^2 + bx + c$ with a positive a value is $f(x) \geq f\left(-\frac{b}{2a}\right)$, or $\left[f\left(-\frac{b}{2a}\right), \infty\right)$; the range of a quadratic function written in general form with a negative a value is $f(x) \leq f\left(-\frac{b}{2a}\right)$, or $\left(-\infty, f\left(-\frac{b}{2a}\right)\right]$.

The range of a quadratic function written in standard form $f(x) = a(x - h)^2 + k$ with a positive a value is $f(x) \geq k$; the range of a quadratic function written in standard form with a negative a value is $f(x) \leq k$.

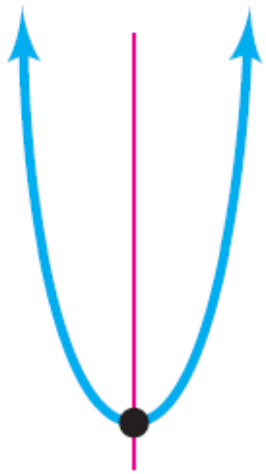
Example 1

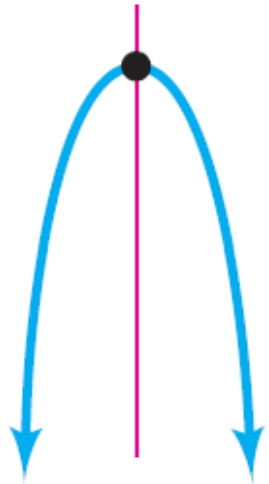
Write the quadratic function $f(x) = x^2 - 4x + 4$ in standard form. Identify the vertex, domain and range of the function.

Solution:

Parabola Orientation

All graphs of quadratic functions of the form $f(x) = ax^2 + bx + c$ are parabolas that open upward or downward.





Parabola Orientation

For the graph of the quadratic function $f(x) = ax^2 + bx + c$, if

- $a > 0$, the parabola opens upward 
- $a < 0$, the parabola opens downward 

Example 2

Determine whether each parabola opens upward or downward:

(a) $f(x) = -3x^2 + 2x - 4$ (b) $f(x) = 6x^2 + 7x - 9$

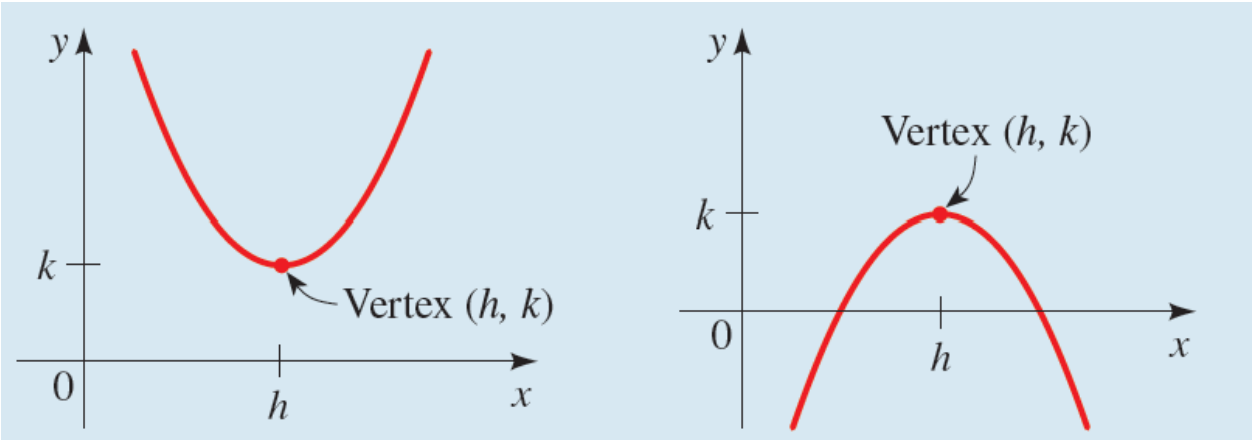
Solution:

2 – Find the Axis of Symmetry and Vertex of a Parabola

Axis of Symmetry and Vertex of a Parabola

The graph of the function $f(x) = ax^2 + bx + c$ is a parabola where:

- the axis of symmetry is the vertical line $x = -\frac{b}{2a}$.
- the vertex is a point on the axis of symmetry, so its x-coordinate is $-\frac{b}{2a}$.
- the y-coordinate of the vertex is found by substituting $x = -\frac{b}{2a}$ into the quadratic equation.



Example 3

For the graph of $f(x) = 3x^2 - 6x + 2$ find:

(a) the axis of symmetry (b) the vertex (h, k) and (c) the domain and range.

Solution:

Example 4 – Finding the Inverse of a Quadratic Function When the Restriction Is Not Specified

Restrict the domain and then find the inverse of $f(x) = (x-2)^2 - 3$.

Solution:

3 – Find the Intercepts of a Parabola

When we graphed linear equations, we often used the $x-$ and $y-$intercepts to help us graph the lines. Finding the coordinates of the intercepts will help us to graph parabolas, too.

Find the Intercepts of a Parabola

To find the intercepts of a parabola whose function is $f(x) = ax^2 + bx + c$:

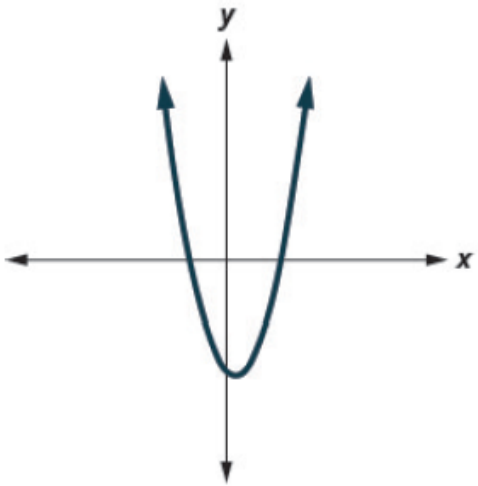
***y*-intercept**	***x*-intercepts**
Let $x = 0$ and solve for $f(x)$.	Let $f(x) = 0$ and solve for x.

Example 5

Find the intercepts of the parabola whose function is $f(x) = x^2 - 2x - 8$.

Solution:

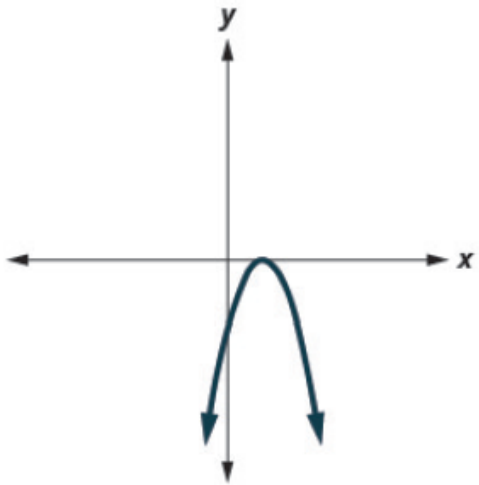
We can use the discriminant to tell us how many $x-$intercepts there are on the graph.



$b^2 - 4ac > 0$

Two solutions

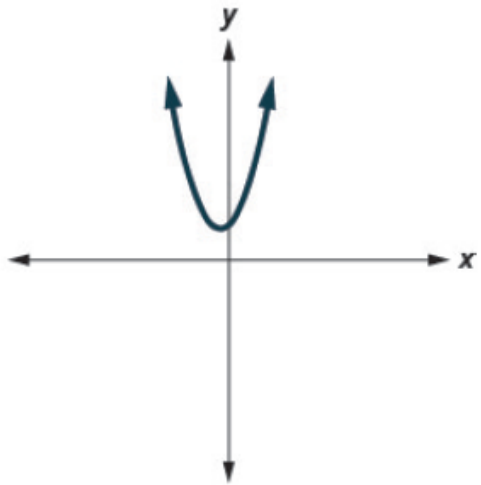
Two x-intercepts



$b^2 - 4ac = 0$

One solution

One x-intercept



$b^2 - 4ac < 0$

No real solution

No x-intercept

Example 6

Find the intercepts of the parabola whose function is $f(x) = 5x^2 + x + 4$.

Solution:

4 – Graph Quadratic Functions Using Properties

HOW TO

To graph a quadratic function using properties.

Step 1. Determine whether the parabola opens upward or downward.
Step 2. Find the equation of the axis of symmetry.
Step 3. Find the vertex.
Step 4. Find the y-intercept. Find the point symmetric to the y-intercept across the axis of symmetry.
Step 5. Find the x-intercepts. Find additional points if needed.
Step 6. Graph the parabola.

Example 7

Graph the function $f(x) = -x^2 + 6x - 9$ by using its properties. State the domain and range on your graph.

Solution:

Example 8

Graph the function $f(x) = x^2 + 4x + 5$ by using its properties. State the domain and range on your graph.

Solution:

Example 9

Graph the function $f(x) = x^2 + 4x + 5$ by using its properties. State the domain and range on your graph.

Solution:

5 – Solve Maximum and Minimum Applications

MINIMUM OR MAXIMUM VALUES OF A QUADRATIC FUNCTION

The ***y*-coordinate of the vertex** of the graph of a quadratic function is the

- *minimum* value of the quadratic equation if the parabola opens *upward*.
- *maximum* value of the quadratic equation if the parabola opens *downward*.

MAXIMUM OR MINIMUM VALUE OF A QUADRATIC FUNCTION

The maximum or minimum value of a quadratic function $f(x) = ax^2 + bx + c$ occurs at

$$x = -\frac{b}{2a}$$

If $a > 0$, then the **minimum value** is $f\left(-\frac{b}{2a}\right)$.

If $a < 0$, then the **maximum value** is $f\left(-\frac{b}{2a}\right)$.

Example 10

Find the minimum or maximum value of the quadratic function $f(x) = x^2 + 2x - 8$.

Solution: