4.1 – Linear Functions

Learning Objectives:

- Find the slope of a linear function
- Graph a linear function using its slope and intercept
- Graph and interpret applications of slope–intercept
- Use slopes to identify parallel and perpendicular lines

Linear Function

A **linear function** is a function whose graph is a line. Linear functions can be written in the **slope-intercept** form of a line

$$f(x) = mx + b$$

where b is the initial or starting value of the function (when input, $x = 0$),and m is the constant rate of change, or slope of the function. The $y-$intercept is at $(0, b)$.

1 – Find the Slope of a Linear Function

Calculate Slope

The slope, or rate of change, of a function m can be calculated according to the following:

$$m = \frac{\text{change in output (rise)}}{\text{change in input (run)}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

where x_1 and x_2 are input values, y_1 and y_2 are output values.

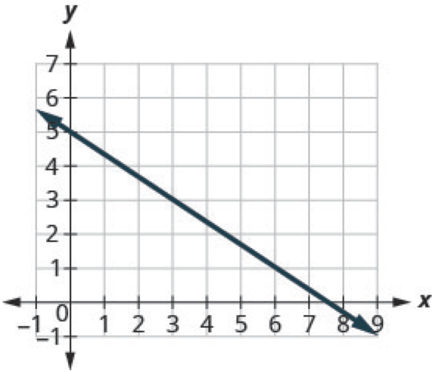
HOW TO

Find the slope of a line from its graph using $m = \frac{\text{rise}}{\text{run}}$.

Step 1. Locate two points on the line whose coordinates are integers.
Step 2. Starting with one point, sketch a right triangle, going from the first point to the second point.
Step 3. Count the rise and the run on the legs of the triangle.
Step 4. Take the ratio of rise to run to find the slope: $m = \frac{\text{rise}}{\text{run}}$.

Example 1

Find the slope of the line shown.

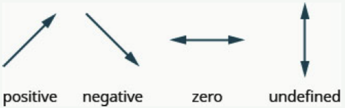


Slope of a Horizontal and Vertical Line

The slope of a horizontal line, $y = b$, is 0.

The slope of a vertical line, $x = a$, is undefined.

Quick Guide to the Slopes of Lines



Example 2

Find the slope of each line: (a) $x = 8$ and (b) $y = -5$.

SOLUTION:

Example 3

Use the slope formula to find the slope of the line through the points $(-2, -3)$ and $(-7, 4)$.

SOLUTION:

2 – Graph a Linear Function Using its Slope and Intercept

Graphical Interpretation of a Linear Function

In the equation $f(x) = mx + b$

- b is the y-intercept of the graph and indicates the point $(0, b)$ at which the graph crosses the y-axis.
- m is the slope of the line and indicates the vertical displacement (rise) and horizontal displacement (run) between each successive pair of points. Recall the formula for the slope:

$$m = \frac{\text{change in output (rise)}}{\text{change in input (run)}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 4

Identify the slope and y-intercept of the line from the equation:
(a) $y = -\frac{4}{7}x - 2$ and (b) $x + 3y = 9$

SOLUTION:

Example 5

Graph the function $f(x) = -x + 4$ using its slope and $y-$intercept.

SOLUTION:

3 – Graph and Interpret Applications of Slope–Intercept

Many real-world applications are modeled by linear equations. We will take a look at a few applications here so you can see how equations written in slope–intercept form relate to real world situations.

Usually, when a linear equation models uses real-world data, different letters are used for the variables, instead of using only x and y. The variable names remind us of what quantities are being measured.

Also, we often will need to extend the axes in our rectangular coordinate system to bigger positive and negative numbers to accommodate the data in the application.

Example 6

The function $F(C) = \frac{9}{5}C + 32$ is used to convert temperatures, C, on the Celsius scale to temperatures, F, on the Fahrenheit scale.

(a) Find the Fahrenheit temperature for a Celsius temperature of 0.

(b) Find the Fahrenheit temperature for a Celsius temperature of 20.

(c) Interpret the slope and F-intercept of the equation.

(d) Graph the function.

Solution to Example 6

The cost of running some types of business has two components—a fixed cost and a variable cost.

The fixed cost is always the same regardless of how many units are produced. This is the cost of rent, insurance, equipment, advertising, and other items that must be paid regularly.

The variable cost depends on the number of units produced. It is for the material and labor needed to produce each item.

Example 7

Sam drives a delivery van. The function $C(m) = 0.5m + 60$ models the relation between his weekly cost, C, in dollars and the number of miles, m, that he drives.

(a) Find Sam's cost for a week when he drives 0 miles.

(b) Find the cost for a week when he drives 250 miles.

(c) Interpret the slope and C-intercept of the equation.

(d) Graph the equation.

Solution to Example 7

4 – Use Slopes to Identify Parallel and Perpendicular Lines

Parallel and Perpendicular Lines

Two lines are **parallel lines** if they do not intersect. The slopes of the lines are the same.

$$f(x) = m_1 x + b_1 \text{ and } g(x) = m_2 x + b_2 \text{ are parallel if and only if } m_1 = m_2$$

If and only if $b_1 = b_2$ and $m_1 = m_2$, we say the lines coincide. Coincident lines are the same line.

Two lines are **perpendicular lines** if they intersect to form a right angle.

$$f(x) = m_1 x + b_1 \text{ and } g(x) = m_2 x + b_2 \text{ are perpendicular if and only if}$$

$$m_1 m_2 = -1, \text{so } m_2 = -\frac{1}{m_1}$$

Example 8

Find the equation of a line perpendicular to $f(x) = 3x + 3$ that passes through the point $(3, 0)$.

SOLUTION:

Example 9

Find a line parallel to the graph of $f(x) = 3x + 6$ that passes through the point $(3, 0)$.

SOLUTION:

Example 10

A line passes through the points $(-2, 6)$ and $(4, 5)$. Find the equation of a perpendicular line that passes through the point $(4, 5)$.

SOLUTION: