3.5 – Transformation of Functions

Learning Objectives

1. Graph functions using vertical and horizontal shifts.
2. Graph functions using reflections about the x-axis and the y-axis.
3. Determine whether a function is even, odd, or neither from its graph.

1 – Graphing Functions Using Vertical and Horizontal Shifts

Often when given a problem, we try to model the scenario using mathematics in the form of words, tables, graphs, and equations. One method we can employ is to adapt the basic graphs of the toolkit functions to build new models for a given scenario. There are systematic ways to alter functions to construct appropriate models for the problems we are trying to solve.

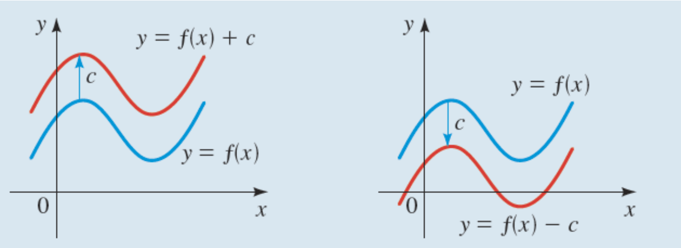
Identifying Vertical Shifts

Vertical Shift: Given the function $f(x)$, a new function $g(x) = f(x) + c$, where c is a constant, is a **vertical shift** of the function $f(x)$.

All the output values change by c units.

If c is positive, the graph will shift up.

If c is negative, the graph will shift down.



Example 1 – Vertical Shifts of Graphs

Use the graph of $f(x) = x^2$ to sketch the graph of each function. Find the domain and range of f, g and h.
(a) $g(x) = x^2 + 3$ (b) $h(x) = x^2 - 2$
Solution:

Example 2 – Shifting a Tabular Function Vertically

A function $f(x)$ is given in the table below. Create a table for $g(x) = f(x) - 3$.

x	2	4	6	8
$f(x)$	1	3	7	11

Solution:

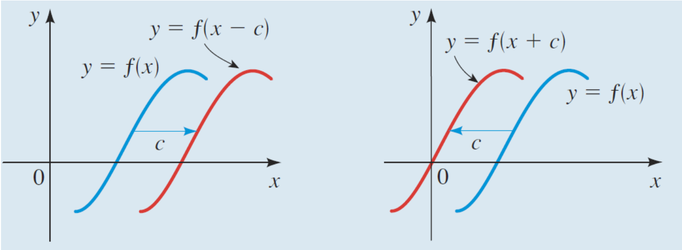
Identifying Horizontal Shifts

Horizontal Shift: Given the function $f(x)$, a new function $g(x) = f(x - c)$, where c is a constant, is a **horizontal shift** of the function $f(x)$.

If c is positive, the graph will shift right by c units.

If c is negative, the graph will shift left by c units.

Note: When c is negative we have $f(x - (-c)) = f(x + c)$.



Example 3 – Horizontal Shifts of Graphs

Use the graph of $f(x) = \sqrt{x}$ to sketch the graph of each function. Find the domain and range of f, g and h.
(a) $g(x) = \sqrt{x-2}$ (b) $h(x) = \sqrt{x+4}$
Solution:

Example 4 – Shifting a Tabular Function Horizontally

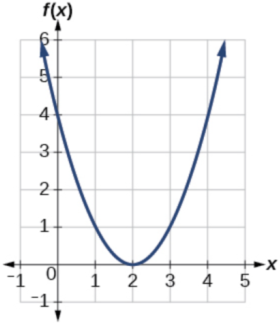
A function $f(x)$ is given in the table below. Create a table for $g(x) = f(x-3)$.

x	2	4	6	8
$f(x)$	1	3	7	11

Solution:

Example 5 – Identifying a Horizontal Shift of a Toolkit Function

The graph below represents a transformation of the toolkit function $f(x) = x^2$. Relate this new function $g(x)$ to $f(x) = x^2$, and then find a formula for $g(x)$.



Combining Vertical and Horizontal Shifts

Given a function and both a vertical and a horizontal shift, sketch the graph.

1. Identify the vertical and horizontal shifts from the formula.
2. The vertical shift results from a constant added to the output. Move the graph up for a positive constant and down for a negative constant.
3. The horizontal shift results from a constant added to the input. Move the graph left for a positive constant and right for a negative constant.
4. Apply the shifts to the graph in either order.

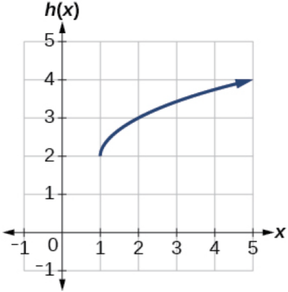
Example 6 – Graphing Combined Vertical and Horizontal Shifts

Given $f(x) = |x|$, sketch a graph of $h(x) = f(x+1) - 3$. Find the domain and range of h.

Solution:

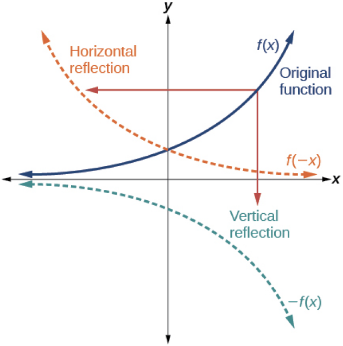
Example 7 – Identifying Combined Vertical and Horizontal Shifts

Write a formula for the graph shown below, which is a transformation of the toolkit function $f(x) = \sqrt{x}$.



2 – Graphing Functions Using Reflections about the Axes

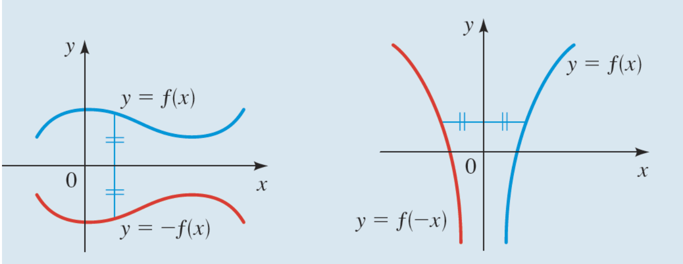
Another transformation that can be applied to a function is a reflection over the $x-$ or $y-$axis. A **vertical reflection** reflects a graph vertically across the $x-$axis, while a **horizontal reflection** reflects a graph horizontally across the $y-$axis.



Reflections

Given $f(x)$, a new function $g(x) = -f(x)$ is a **vertical reflection** of the function $f(x)$, sometimes called a reflection over the $x-$axis.

Given $f(x)$, a new function $g(x) = f(-x)$ is a **horizontal reflection** of the function $f(x)$, sometimes called a reflection over the $y-$axis.



Example 8 – Reflecting Graphs

Use the graph of $f(x) = \sqrt{x}$ to sketch the graph of each function. Find the domain and range of f, g and h.
(a) $g(x) = -\sqrt{x}$ (b) $h(x) = \sqrt{-x}$
Solution:

3 – Determining Even and Odd Functions

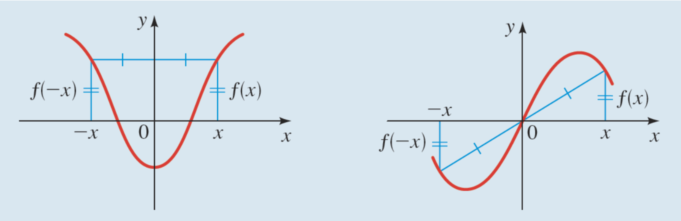
Even and Odd Functions

A function is called an **even function** if for every input x
$f(x) = f(-x)$.

The graph of an even function is symmetric about the $y-$axis.

A function is called an **odd function** if for every input x
$f(x) = -f(x)$.

The graph of an odd function is symmetric about the origin.



HOW TO

Given the formula for a function, determine if the function is even, odd, or neither.

1. Determine whether the function satisfies $f(x) = f(-x)$. If it does, it is even.
2. Determine whether the function satisfies $f(x) = -f(-x)$. If it does, it is odd.
3. If the function does not satisfy either rule, it is neither even nor odd.

Example 9 – Determining whether a Function Is Even, Odd, or Neither

Determine whether the function $f(x) = x^3 + 2x$ is even, odd, or neither.

Solution: