Learning Objectives

1. Combine functions using algebraic operations.
2. Create a new function by composition of functions.
3. Evaluate composite functions.
4. Find the domain of a composite function.
5. Decompose a composite function into its component functions.

1 – Combining Functions Using Algebraic Operations

Algebra of Functions:
Let f and g be functions. Then the functions $f+g$, $f-g$, fg, and f/g are defined as follows.

1. $(f+g)(x) = f(x) + g(x)$
2. $(f-g)(x) = f(x) - g(x)$
3. $(fg)(x) = f(x)g(x)$
4. $\left(\frac{f}{g}\right) = \frac{f(x)}{g(x)}$, $g(x) \neq 0$

Example 1 – Performing Algebraic Operations on Functions

Let $f(x) = x - 1$ and $g(x) = x^2 - 1$. Find and simplify $(g - f)(x)$ and $\left(\frac{g}{f}\right)(x)$.

Solution:

2 – Create a Function by Composition of Functions

Performing algebraic operations on functions combines them into a new function, but we can also create functions by composing functions.

The process of combining functions so that the output of one function becomes the input of another is known as a **composition of functions**.

Composition of Functions

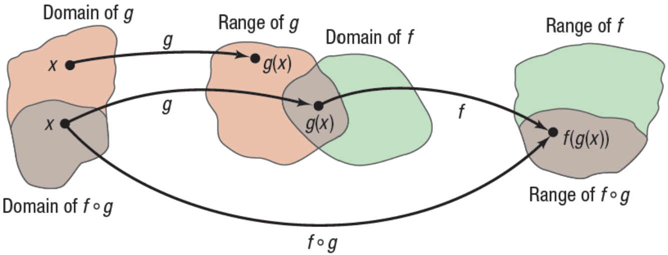
When the output of one function is used as the input of another, we call the entire operation a composition of functions. For any input x and functions f and g this action defines a **composite function**, which we write as $f \circ g$ such that

$$(f \circ g)(x) = f(g(x))$$

The domain of the composite function $f \circ g$ is a ll x such that x is in the domain of g and $g(x)$ is in the domain of f.

Note: The product of functions fg is *not* the same as the function composition $f(g(x))$, because in general $f(x)g(x) \neq f(g(x))$.

Look carefully at figure below. Only those values of x in the domain of g for which $g(x)$ is in the domain of f can be in the domain of $f \circ g$. The reason is if $g(x)$ is not in the domain of f, then $f(g(x))$ is not defined.



Example 2 – Finding the Composition of Functions

Let $f(x) = 2x + 1$ and $g(x) = 3 - x$. Find the functions $f \circ g$ and $g \circ f$.

Solution:

Example 3 – Interpreting Composite Functions

The function $c(s)$ gives the number of calories burned completing s sit-ups, and $s(t)$ gives the number of sit-ups a person can complete in t minutes. Interpret $c(s(3))$.

Solution:

3 – Evaluating Composite Functions

Once we compose a new function from two existing functions, we need to be able to evaluate it for any input in its domain. We will do this with specific numerical inputs for functions expressed as tables, graphs, and formulas and with variables as inputs to functions expressed as formulas. In each case, we evaluate the inner function using the starting input and then use the inner function's output as the input for the outer function.

Example 4 – Using a Table to Evaluate a Composite Function

Use the table below to find $f(g(3))$ and $g(f(3))$.

x	$f(x)$	$g(x)$
1	6	3
2	8	5
3	3	2
4	1	7

Evaluating Composite Functions Using Graphs

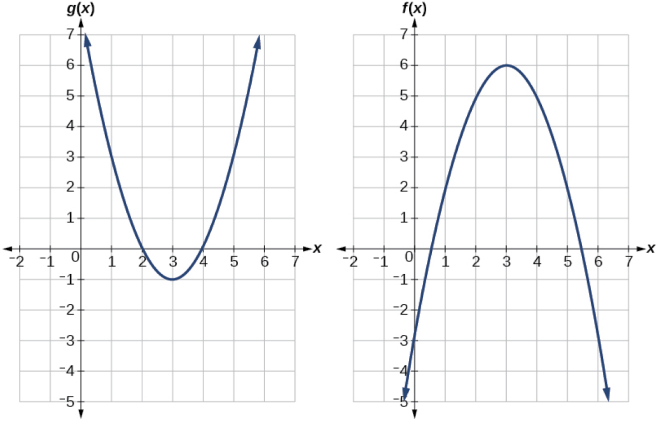
HOW TO

Given a composite function and graphs of its individual functions, evaluate it using the information provided by the graphs.

1. Locate the given input to the inner function on the x- axis of its graph.
2. Read off the output of the inner function from the y- axis of its graph.
3. Locate the inner function output on the x- axis of the graph of the outer function.
4. Read the output of the outer function from the y- axis of its graph. This is the output of the composite function.

Example 5 – Using a Graph to Evaluate a Composite Function

Use the graphs below to evaluate $f(g(1))$.



(a) (b)

Evaluating Composite Functions Using Formulas

HOW TO

Given a formula for a composite function, evaluate the function.

1. Evaluate the inside function using the input value or variable provided.
2. Use the resulting output as the input to the outside function.

Example 6 – Evaluating a Composition of Functions Expressed as Formulas with a Numerical Input

Given $f(t) = t^2 - t$ and $h(x) = 3x + 2$, find $f(h(1))$.

Solution:

Domain of a Composite Function
The domain of a composite function $f(g(x))$ is the set of those inputs x in the domain of g for which $g(x)$ is in the domain of f.

HOW TO

Given a function composition $f(g(x))$, determine its domain.

1. Find the domain of g.
2. Find the domain of f.
3. Find those inputs x in the domain of g for which $g(x)$ is in the domain of f. That is, exclude those inputs x from the domain of g for which $g(x)$ is not in the domain of f. The resulting set is the domain of $f \circ g$.

Example 7 – Finding the Domain of a Composite Function

Find the domain of

$$(f \circ g)(x) \text{ where } \qquad f(x) = \frac{5}{x-1} \text{ and } g(x) = \frac{4}{3x-2}$$

Solution:

Example 8 – Finding the Domain of a Composite Function Involving Radicals

Find the domain of

$$(f \circ g)(x) \quad \text{where} \qquad f(x) = \sqrt{x+2} \quad \text{and} \quad g(x) = \sqrt{3-x}$$

Solution:

5 – Decomposing a Composite Function into its Component Functions

In some cases, it is necessary to decompose a complicated function. In other words, we can write it as a composition of two simpler functions. There may be more than one way to decompose a composite function, so we may choose the decomposition that appears to be most expedient.

Example 9 – Decomposing a Function

Write $f(x) = \sqrt{5 - x^2}$ as the composition of two functions.
Solutions: