3.3 - Rates of Change and Behavior of Graphs

Learning Objectives

1. Find the average rate of change of a function.
2. Use a graph to determine where a function is increasing, decreasing, or constant.
3. Use a graph to locate local maxima and local minima.
4. Use a graph to locate the absolute maximum and absolute minimum.

1 – Finding the Average Rate of Change of a Function

The **Average Rate of Change** between two input values is the total change of the function values (output values) divided by the change in the input values.

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

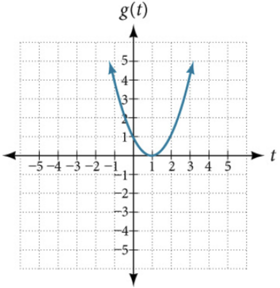
HOW TO

Given the value of a function at different points, calculate the average rate of change of a function for the interval between two values x_1 and x_2.

1. Calculate the difference $y_2 - y_1 = \Delta y$.
2. Calculate the difference $x_2 - x_1 = \Delta x$.
3. Find the ratio $\frac{\Delta y}{\Delta x}$.

Example One – Computing Average Rate of Change from a Graph

Given the function $g(t)$ shown below, find the average rate of change on the interval $[-1, 2]$.



Example Two – Computing Average Rate of Change from a Table

After picking up a friend who lives 10 miles away and leaving on a trip, Anna records her distance from home over time. The values are shown in the table below. Find her average speed over the first 6 hours.

t (hours)	0	1	2	3	4	5	6	7
$D(t)$ (miles)	10	55	90	153	214	240	292	300

Solution:

Example Three – Computing Average Rate of Change for a Function Expressed as a Formula

Find the average rate of change of $f(x) = x^2 - \frac{1}{x}$ on the interval $[2, 4]$.

Solution:

Example Four – Finding an Average Rate of Change as an Expression

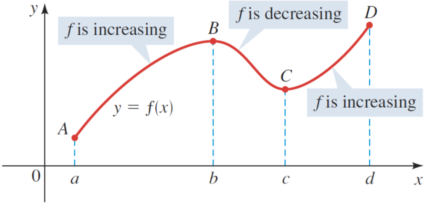
Find the average rate of change of $g(t) = t^2 + 3t + 1$ on the interval $[0, a]$.

Solution:

2 – Using a Graph to Determine Where a Function is Increasing, Decreasing, or Constant

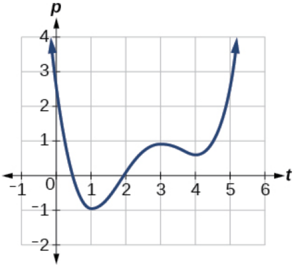
It is very useful to know where the graph of a function rises and where it falls.

The graph shown below rises, falls, then rises again as we move from left to right: It rises from A to B, falls from B to C, and rises again from C to D. The function f is said to be *increasing* when its graph rises and *decreasing* when its graph falls.



Example Five – Finding Increasing and Decreasing Intervals on a Graph

Given the function $p(t)$, shown below. Find the intervals on which the function is increasing.



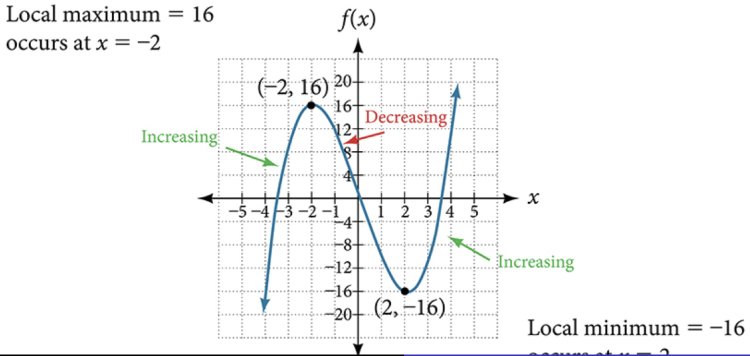
3 – Use a Graph to locate Local Maxima and Local Minima

Local Minima and Local Maxima

A function f is an **increasing function** on an open interval if $f(b) > f(a)$ for any two input values a and b in the given interval where $b > a$.

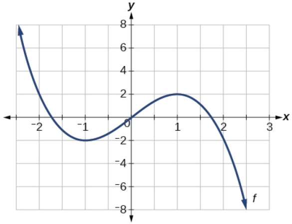
A function f is a **decreasing function** on an open interval if $f(b) < f(a)$ for any two input values a and b in the given interval where $b > a$.

A function f has a local maximum at $x = b$ if there exists an interval (a, c) with $a < b < c$ such that, for any x in the interval (a, c), $f(x) \leq f(b)$. Likewise, f has a local minimum at $x = b$ if there exists an interval (a, c) with $a < b < c$ such that, for any x in the interval (a, c), $f(x) \geq f(b)$.

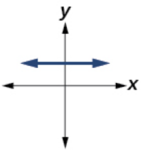
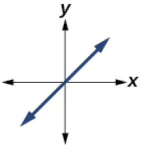
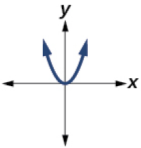


Example Six – Finding Local Maxima and Minima from a Graph

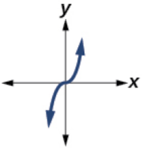
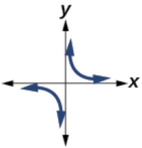
For the function f whose graph is shown below, find all local maxima and minima.



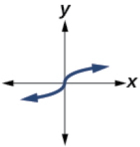
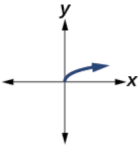
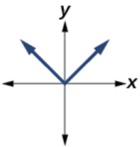
Analyzing the Toolkit Functions for Increasing or Decreasing Intervals

Function	Increasing/Decreasing	Example
Constant Function $f(x) = c$	Neither increasing nor decreasing	
Identity Function $f(x) = x$	Increasing	
Quadratic Function $f(x) = x^2$	Increasing on $(0, \infty)$ Decreasing on $(-\infty, 0)$ Minimum at $x = 0$	

Analyzing the Toolkit Functions for Increasing or Decreasing Intervals

Function	Increasing/Decreasing	Example
Cubic Function $f(x) = x^3$	Increasing	
Reciprocal $f(x) = \frac{1}{x}$	Decreasing $(-\infty, 0) \cup (0, \infty)$	

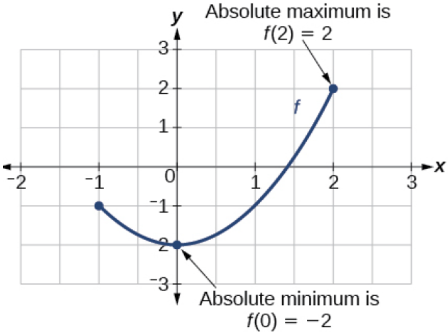
Analyzing the Toolkit Functions for Increasing or Decreasing Intervals

Function	Increasing/Decreasing	Example
Cube Root $f(x) = \sqrt[3]{x}$	Increasing	
Square Root $f(x) = \sqrt{x}$	Increasing on $(0, \infty)$	
Absolute Value $f(x) = \lvert x \rvert$	Increasing on $(0, \infty)$ Decreasing on $(-\infty, 0)$	

4 – Use A Graph to Locate the Absolute Maximum and Absolute Minimum

There is a difference between locating the highest and lowest points on a graph in a region around an open interval (locally) and locating the highest and lowest points on the graph for the entire domain. The $y-$coordinates (output) at the highest and lowest points are called the absolute maximum and absolute minimum, respectively.

To locate absolute maxima and minima from a graph, we need to observe the graph to determine where the graph attains it highest and lowest points on the domain of the function.



Not every function has an absolute maximum or minimum value. The toolkit function $f(x) = x^3$ is one such function.

Absolute Maxima and Minima

Absolute Maxima and Minima

The **absolute maximum** of f at $x = c$ is $f(c)$ where $f(c) \geq f(x)$ for all x in the domain of f.

The **absolute minimum** of f at $x = d$ is $f(d)$ where $f(d) \leq f(x)$ for all x in the domain of f.

Example Seven – Finding Absolute Maxima and Minima from a Graph

For the function f shown below, find all the absolute maxima and minima.