3.2 – Domain and Range

Learning Objectives

1. Find the domain of a function defined by an equation.
2. Graph piecewise-defined functions.

1 – Finding the Domain of a Function Defined by an Equation

Conventions of Interval Notation:

1. The smallest number from the interval is written first.
2. The largest number in the interval is written second, following a comma.
3. Parentheses, (or), are used to signify that an endpoint value is not included, called exclusive.
4. Brackets, [or], are used to indicate that an endpoint value is included, called inclusive.

Interval Notation

Inequality	Interval Notation	Graph on Number Line	Description
$x > a$	(a, ∞)		x is greater than a
$x < a$	$(-\infty, a)$		x is less than a
$x \geq a$	$[a, \infty)$		x is greater than or equal to a
$x \leq a$	$(-\infty, a]$		x is less than or equal to a
$a < x < b$	(a, b)		x is strictly between a and b
$a \leq x < b$	$[a, b)$		x is between a and b, to include a
$a < x \leq b$	$(a, b]$		x is between a and b, to include b
$a \leq x \leq b$	$[a, b]$		x is between a and b, to include a and b

Example 1 – Finding the Domain of a Function as a Set of Ordered Pairs

Find the domain of the following function:
$\{(2, 10), (3, 10), (4, 20), (5, 30), (6, 40)\}$.
Solution:

Example Two – Finding the Domain of a Polynomial Function

Find the domain of the following function: $f(x) = x^2 - 1$

Solution:

HOW TO

Given a function written in an equation form that includes a fraction, find the domain.

1. Identify the input values.
2. Identify any restrictions on the input. If there is a denominator in the function's formula, set the denominator equal to zero and solve for x . If the function's formula contains an even root, set the radicand greater than or equal to 0, and then solve.
3. Write the domain in interval form, making sure to exclude any restricted values from the domain.

Example Three – Finding the Domain of a Function Involving a Denominator

Find the domain of the following function: $f(x) = \dfrac{x+1}{2-x}$

Solution:

HOW TO

Given a function written in equation form including an even root, find the domain.

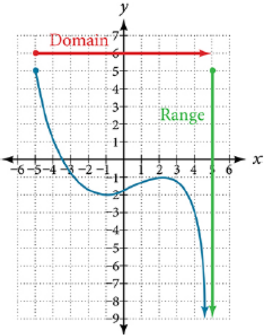
1. Identify the input values.
2. Since there is an even root, exclude any real numbers that result in a negative number in the radicand. Set the radicand greater than or equal to zero and solve for x.
3. The solution(s) are the domain of the function. If possible, write the answer in interval form.

Example Four – Finding the Domain of a Function with an Even Root

Find the domain of the following function: $f(x) = \sqrt{7 - x}$
Solution:

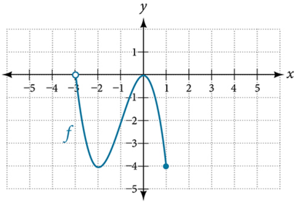
Finding Domain and Range from Graphs

Another way to identify the domain and range of functions is by using graphs. Because the domain refers to the set of possible input values, the domain of a graph consists of all the input values shown on the x-axis. The range is the set of possible output values, which are shown on the y-axis. Keep in mind that if the graph continues beyond the portion of the graph we can see, the domain and range may be greater than the visible values.



Example Five – Finding Domain and Range from a Graph

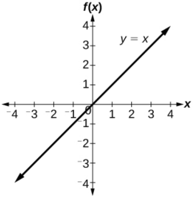
Find the domain and range of the function whose graph is shown below.



Solution:

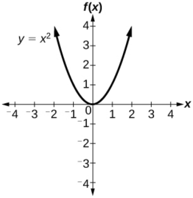
Domain and Range of the Toolkit Functions

Identity



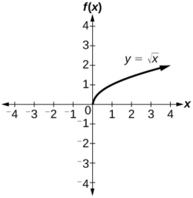
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$

Square



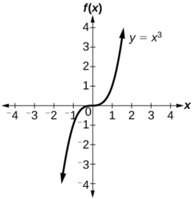
Domain: $(-\infty, \infty)$
Range: $[0, \infty)$

Square Root



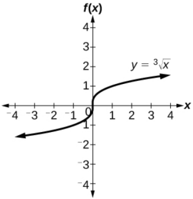
Domain: $[0, \infty)$
Range: $[0, \infty)$

Cubic



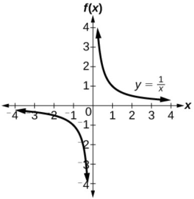
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$

Cube Root



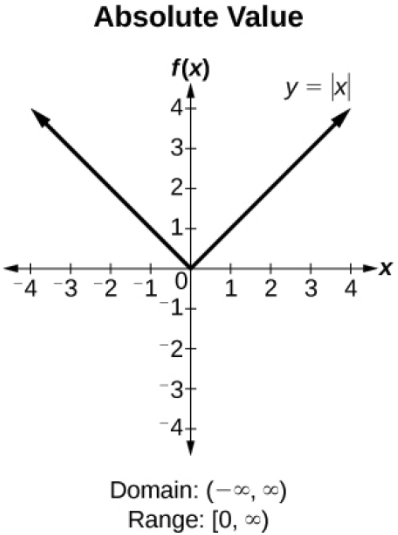
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$

Reciprocal



Domain: $(-\infty, 0) \cup (0, \infty)$
Range: $(-\infty, 0) \cup (0, \infty)$

Domain and Range of the Toolkit Functions



HOW TO

Given the formula for a function, determine the domain and range.

1. Exclude from the domain any input values that result in division by zero.
2. Exclude from the domain any input values that have nonreal (or undefined) number outputs.
3. Use the valid input values to determine the range of the output values.
4. Look at the function graph and table values to confirm the actual function behavior.

Example Six – Finding the Domain and Range

Find the domain and range of the following functions:

(a) $f(x) = 2x^3 - x$ (b) $f(x) = \dfrac{2}{x+1}$ (c) $f(x) = 2\sqrt{x+4}$

Solution:

2 – Graph Piecewise-Defined Functions

A **piecewise function** is a function in which more than one formula is used to define the output. Each formula has its own domain, and the domain of the function is the union of all these smaller domains.

$$f(x) = \begin{cases} \text{formula 1} & \text{if } x \text{ is in domain 1} \\ \text{formula 2} & \text{if } x \text{ is in domain 2} \\ \text{formula 3} & \text{if } x \text{ is in domain 3} \end{cases}$$

Example Seven - Evaluating a Piecewise Functions

Let

$$f(x) = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Find $f(0), f(-1), f(1)$ and $f(0.5)$.

Solution:

Example Eight - Working with a Piecewise Function

A cell phone company uses the function below to determine the cost, C in dollars for g gigabytes of data transfer.

$$C(g) = \begin{cases} 25 & \text{if } 0 < g < 2 \\ 25 + 10(g - 2) & \text{if } g \geq 2 \end{cases}$$

Find the cost of using 1.5 gigabytes of data and the cost of using 4 gigabytes of data

Solution:

HOW TO

Given a piecewise function, sketch a graph.

1. Indicate on the x-axis the boundaries defined by the intervals on each piece of the domain.
2. For each piece of the domain, graph on that interval using the corresponding equation pertaining to that piece. Do not graph two functions over one interval because it would violate the criteria of a function.

Example Nine - Graphing a Piecewise Function

Sketch a graph of the function

$$f(x) = \begin{cases} x^2 & \text{if } x \leq 1 \\ 2x+1 & \text{if } x > 1 \end{cases}$$

Solution:

Example Ten - Graphing a Piecewise Function

Sketch a graph of the function

$$f(x) = \begin{cases} x^2 & \text{if } x \leq 1 \\ 3 & \text{if } 1 < x \leq 2 \\ x & \text{if } x > 2 \end{cases}$$

Solution: