3.1 – Functions and Function Notation

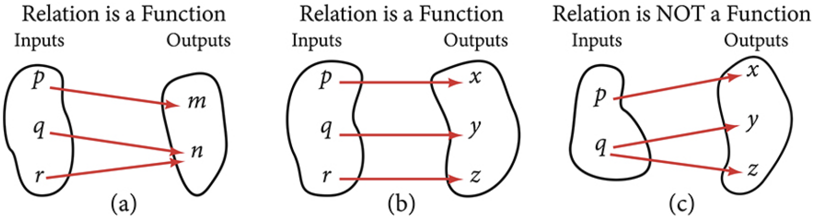
Learning Objectives

1. Determine whether a relation represents a function
2. Find the value of a function
3. Determine whether a function is one-to-one
4. Use the vertical line test to identify functions
5. Develop a Toolkit of Functions

1 – Determining Whether a Relation Represents a Function

Relation: A relation is a set of ordered pairs. The set of the first components of each ordered pair is called the **domain** and the set of the second components of each ordered pair is called the **range**.

Function: A function is a relation in which each possible input value leads to exactly one output value. We say "the output is a function of the input." The **input** values make up the **domain**, and the output values make up the **range**.



HOW TO: Determine whether relation is a function

HOW TO

Given a relationship between two quantities, determine whether the relationship is a function.

1. Identify the input values.
2. Identify the output values.
3. If each input value leads to only one output value, classify the relationship as a function. If any input value leads to two or more outputs, do not classify the relationship as a function.

Example 1 – Determining If Menu Price Lists Are Functions

The coffee shop menu shown below, consists of items and their prices.

Menu

Item	Price
Plain Donut	1.49
Jelly Donut	1.99
Chocolate Donut	1.99

a) Is the price a function of the item?
b) Is the item a function of the price?

Solution:

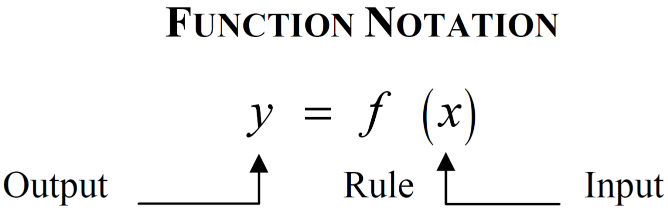
Using Function Notation

Function Notation

For the function $y = f(x)$

f is the name of the function
x is the domain value
$f(x)$ is the range value y corresponding to the value x

We read $f(x)$ as f of x or the value of f at x.



Example 2 – Using Function Notation for Days in a Month

Use function notation to represent a function whose input is the name of a month and output is the number of days in that month. Assume that the domain does not include leap years.

Solution:

Example 3 – Interpreting Function Notation

A function $N = f(y)$ gives the number of police officers, N, in a town in year y. What does $f(2005) = 300$ represent?

Solution:

Representing Functions Using Tables

A common method of representing functions is in the form of a table. The table rows or columns display the corresponding input and output values. In some cases, these values represent all we know about the relationship; other times, the table provides a few select examples from a more complete relationship.

HOW TO

Given a table of input and output values, determine whether the table represents a function.

1. Identify the input and output values.
2. Check to see if each input value is paired with only one output value. If so, the table represents a function.

Example 4 – Identifying Tables that Represent Functions

Which table, Table 6, Table 7, or Table 8, represents a function?

Input	Output
2	1
5	3
8	6

Table 6

Input	Output
–3	5
0	1
4	5

Table 7

Input	Output
1	0
5	2
5	4

Table 8

Solution:

2 – Find the value of a Function

HOW TO

Given the formula for a function, evaluate.

1. Substitute the input variable in the formula with the value provided.
2. Calculate the result.

Example 5 – Evaluating Functions at Specific Values

Evaluate $f(x) = x^2 + 3x - 4$ at:

(a) 2 (b) a (c) $a + h$ (d) Now evaluate $\frac{f(a+h)-f(a)}{h}$

Solution:

Difference Quotient

The expression in example 5 (d) is called the difference quotient of f. Difference quotients occur frequently in calculus.

Three-Step Process for Finding $\dfrac{f(x+h)-f(x)}{h}$

1. Compute $f(x + h)$.

2. Form the difference $f(x + h) - f(x)$.

3. Form the quotient $\dfrac{f(x + h) - f(x)}{h}$.

Example 6 – Solving Functions

Given the function $h(p) = p^2 + 2p$, solve for $h(p) = 3$.

Solution:

Evaluating Functions Expressed in Formulas

HOW TO

Given a function in equation form, write its algebraic formula.

1. Solve the equation to isolate the output variable on one side of the equal sign, with the other side as an expression that involves *only* the input variable.
2. Use all the usual algebraic methods for solving equations, such as adding or subtracting the same quantity to or from both sides, or multiplying or dividing both sides of the equation by the same quantity.

Note: It is important to note that not every relationship expressed by an equation can also be expressed as a function with a formula.

Example 7 – Finding an Equation of a Function

Express the relationship $2n + 6p = 12$ as a function $p = f(n)$, if possible.

Solution:

Evaluating a Function Given in Tabular Form

HOW TO

Given a function represented by a table, identify specific output and input values.

1. Find the given input in the row (or column) of input values.
2. Identify the corresponding output value paired with that input value.
3. Find the given output values in the row (or column) of output values, noting every time that output value appears.
4. Identify the input value(s) corresponding to the given output value.

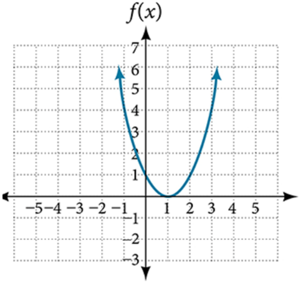
Example 8 – Evaluating and Solving a Tabular Function

Using the table below,

n	1	2	3	4	5
$g(n)$	8	6	7	6	8

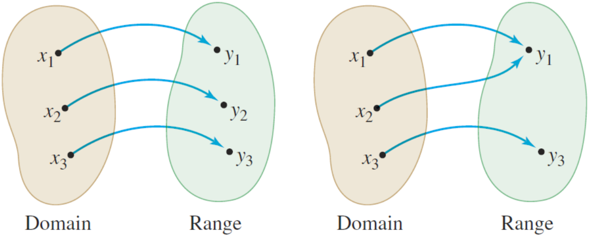
a) Evaluate $g(3)$.
b) Solve $g(n) = 6$.
Solution:

Using the graph below,



a) Evaluate $f(2)$.
b) Solve $f(x) = 4$.
Solution:

One-to-One Function: A one-to-one function is a function in which each output value corresponds to exactly one input value.



Example 10 – Determining Whether a Relationship Is a One-to-One Function

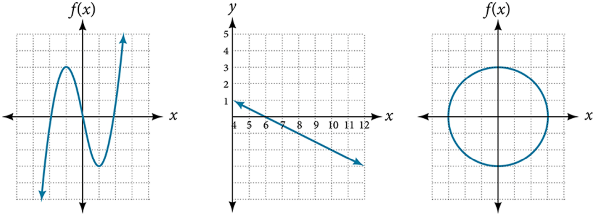
Is the area of a circle a function of its radius? If yes, is the function one-to-one?

Solution:

4 – Use the Vertical Line Test To Identify Functions

Vertical Line Test: A set of points in a rectangular coordinate system is the graph of a function if every vertical line intersects the graph in at most one point. If any vertical line intersects the graph in more than one point, the graph does not represent a function.

Example 11 – Applying the VLT
Which of the graphs below represents a function $y = f(x)$?



(a) (b) (c)

Using the Horizontal Line Test

Once we have determined that a graph defines a function, an easy way to determine if it is a one-to-one function is to use the **horizontal line test**. Draw horizontal lines through the graph. If any horizontal line intersects the graph more than once, then the graph does not represent a one-to-one function.

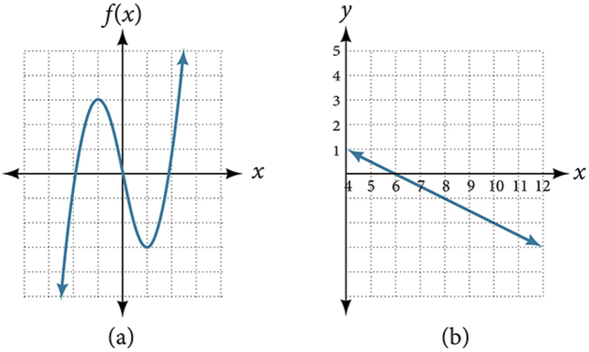
HOW TO

Given a graph of a function, use the horizontal line test to determine if the graph represents a one-to-one function.

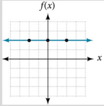
1. Inspect the graph to see if any horizontal line drawn would intersect the curve more than once.
2. If there is any such line, determine that the function is not one-to-one.

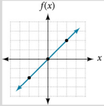
Example 12 – Applying the HLT

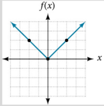
Determine whether the graphs of the functions below are one-to-one functions.

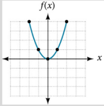


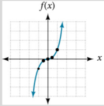
5 – Develop a Toolkit of Functions

<table>
<tr><th colspan="3">Toolkit Functions</th></tr>
<tr><td>Name</td><td>Function</td><td>Graph</td></tr>
<tr><td>Constant</td><td>$f(x) = c$, where c is a constant</td><td>

<table>
<tr><th>x</th><th>f(x)</th></tr>
<tr><td>–2</td><td>2</td></tr>
<tr><td>0</td><td>2</td></tr>
<tr><td>2</td><td>2</td></tr>
</table></td></tr>
<tr><td>Identity</td><td>$f(x) = x$</td><td>

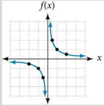
<table>
<tr><th>x</th><th>f(x)</th></tr>
<tr><td>–2</td><td>–2</td></tr>
<tr><td>0</td><td>0</td></tr>
<tr><td>2</td><td>2</td></tr>
</table></td></tr>
<tr><td>Absolute value</td><td>$f(x) = |x|$</td><td>

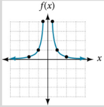
<table>
<tr><th>x</th><th>f(x)</th></tr>
<tr><td>–2</td><td>2</td></tr>
<tr><td>0</td><td>0</td></tr>
<tr><td>2</td><td>2</td></tr>
</table></td></tr>
<tr><td>Quadratic</td><td>$f(x) = x^2$</td><td>

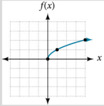
<table>
<tr><th>x</th><th>f(x)</th></tr>
<tr><td>–2</td><td>4</td></tr>
<tr><td>–1</td><td>1</td></tr>
<tr><td>0</td><td>0</td></tr>
<tr><td>1</td><td>1</td></tr>
<tr><td>2</td><td>4</td></tr>
</table></td></tr>
<tr><td>Cubic</td><td>$f(x) = x^3$</td><td>

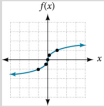
<table>
<tr><th>x</th><th>f(x)</th></tr>
<tr><td>–1</td><td>–1</td></tr>
<tr><td>–0.5</td><td>–0.125</td></tr>
<tr><td>0</td><td>0</td></tr>
<tr><td>0.5</td><td>0.125</td></tr>
<tr><td>1</td><td>1</td></tr>
</table></td></tr>
</table>

Toolkit of Functions Cont.

<table>
<tr><td>Reciprocal</td><td>$f(x) = \frac{1}{x}$</td><td>


<table>
<tr><th>x</th><th>f(x)</th></tr>
<tr><td>–2</td><td>–0.5</td></tr>
<tr><td>–1</td><td>–1</td></tr>
<tr><td>–0.5</td><td>–2</td></tr>
<tr><td>0.5</td><td>2</td></tr>
<tr><td>1</td><td>1</td></tr>
<tr><td>2</td><td>0.5</td></tr>
</table>
</td></tr>
<tr><td>Reciprocal squared</td><td>$f(x) = \frac{1}{x^2}$</td><td>


<table>
<tr><th>x</th><th>f(x)</th></tr>
<tr><td>–2</td><td>0.25</td></tr>
<tr><td>–1</td><td>1</td></tr>
<tr><td>–0.5</td><td>4</td></tr>
<tr><td>0.5</td><td>4</td></tr>
<tr><td>1</td><td>1</td></tr>
<tr><td>2</td><td>0.25</td></tr>
</table>
</td></tr>
<tr><td>Square root</td><td>$f(x) = \sqrt{x}$</td><td>


<table>
<tr><th>x</th><th>f(x)</th></tr>
<tr><td>0</td><td>0</td></tr>
<tr><td>1</td><td>1</td></tr>
<tr><td>4</td><td>2</td></tr>
</table>
</td></tr>
<tr><td>Cube root</td><td>$f(x) = \sqrt[3]{x}$</td><td>


<table>
<tr><th>x</th><th>f(x)</th></tr>
<tr><td>–1</td><td>–1</td></tr>
<tr><td>–0.125</td><td>–0.5</td></tr>
<tr><td>0</td><td>0</td></tr>
<tr><td>0.125</td><td>0.5</td></tr>
<tr><td>1</td><td>1</td></tr>
</table>
</td></tr>
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