Course proposal
An introduction to spectral graph theory

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The purpose of the course is to introduce students to the mathematically rich area of spectral theory associated to finite graphs. On one hand, the topic begins with certain problems in linear algebra, namely the analysis of eigenvalues and eigenfunctions of symmetric, positive definite matrices. As such the problems can be stated in pure mathematical terms. On the other hand, the motivating questions often are derived from other fields of study. Indeed, several introductory textbooks on spectral graph theory include applications to electrical resistance, load balancing in computer networks, and traffic patterns (see, for example, a 1990 article from the New York Times with the catchy title *What if They Closed 42d Street and Nobody Noticed?*).

The goal of the course is to serve as an introduction so then the participants can focus their future studies on their own area of interest. As such, the only prerequisites are a course in linear algebra and some familiarity with programming. Among the possible textbooks are the following.

References

[1] Barlow, M.: *Random Walks and Heat Kernels on Graphs*, London Mathematical Society Lecture Note Series **438** (2017).

[2] Biggs, N. *Algebraic Graph Theory*, Cambridge Mathematical Library (1974).

[3] Brouwer, A., and Haemers, W.: *Spectra of graphs*, Springer Universitext (2010).

[4] Cvetković, D., Rowlinson, P, and Simić, S.: *An Introduction to the Theory of Graph Spectra*, London Mathematical Society Student Texts **75** (2010)

[5] Nica, B.: *A Brief Introduction to Spectral Graph Theory*, European Mathematical Society Textbooks in Mathematics (2018).

The following is a preliminary list of topics which will be studied.

- 1. Review of matrix arithmetic.
- 2. Algebra of symmetric matrices.
- 3. Spectral theory of matrices
- 4. Introduction to graphs.
- 5. Adjacency matrices and Laplacians of graphs.
- 6. Heat kernels on graphs.
- 7. Graph invariants.
- 8. Cayley and Paley graphs.
- 9. Characters of finite abelian groups.
- 10. Determination of graphs by their spectrum.

As time allows, we will provide a discussion as to how certain mathematical questions can be re-interpreted as problems spectral graph theory. Finally, we will give some guide to the current research literature for related questions of interest.

At the end of the course the participants will have seen mathematical ideas from numerous fields, including but not limited to the following: Spectral theory and spectral decomposition, representation theory of finite groups, group actions, arithmetic of finite fields.