

No calculators, cell phones, or other electronic devices allowed.

SHOW ALL WORK. Leave irrational numbers in exact form, e.g. π , $\sqrt{2}$, etc.

PART I. DO ALL 6 PROBLEMS (70 POINTS)

1. (8 points) Evaluate the limits:

$$(a) \quad \lim_{x \rightarrow 0} \frac{\ln(1+x)}{\sin(2x)} \qquad (b) \quad \lim_{x \rightarrow 0^+} (2x)^{x^2}$$

2. (24 points) Integrate and simplify:

$$(a) \quad \int x \ln x \, dx \qquad (b) \quad \int \frac{x+1}{x^2+x-6} \, dx \qquad (c) \quad \int \cot^2 x \cos x \, dx \qquad (d) \quad \int_0^2 \frac{dx}{x^2+4}$$

3. (15 points) Find the derivative $\frac{dy}{dx}$ and simplify:

$$(a) \quad y = \tan^{-1}(e^{x^2}) \qquad (b) \quad y = x^{\sin x} \qquad (c) \quad \sinh(xy) + \ln(y^2) = 5 \cos x$$

4. (7 points) Sketch the graph of the polar equation $r = 4 \sin(3\theta)$ (indicating relevant angles) and compute the area inside one loop.
5. (8 points) Find the length of the portion of the curve $y = (x^4 + 3)/6x$ from $(1, \frac{2}{3})$ to $(3, \frac{14}{3})$.
6. (8 points) Find the volume of the solid generated by revolving about the y -axis the region enclosed by the line $x = 1$, and the graphs of $y = \frac{1}{4}x^3 + 1$ and $y = 1 - x$.

PLEASE TURN OVER, EXAM CONTINUES

PART II. Do 3 COMPLETE PROBLEMS out of 5 (10 points each)

7. (a) Your ability to concentrate on this exam is decaying exponentially. After 1 hour your concentration is $1/4$ of what it was at the beginning of the exam. What fraction of your initial concentration will you have left after $3/2$ hours?
- (b) Find the polynomial plus partial fractions decomposition of

$$\frac{x^3 + x^2}{x^2 + 5x + 6}$$

Note: for full credit you must solve for all coefficients.

8. (a) Use Simpson's (parabolic) rule with 4 subintervals to approximate $\int_1^5 \frac{1}{x+1} dx$. You may leave your answer as a sum of fractions.
- (b) Decide which of the following is of indeterminate form and evaluate it. Explain why the other is not of indeterminate form:

$$(i) \quad \lim_{x \rightarrow \infty} \left(\frac{1}{x}\right)^x \quad (ii) \quad \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x$$

9. A tank in the shape of a right circular cone of height $5ft$ and radius $3ft$, with its circular base on the ground so that its axis is perpendicular to the ground and its vertex is pointing straight up into the air, is filled to a height of $2ft$ with a liquid weighing $4lb/ft^3$. Find the work done in pumping the liquid out through the top vertex of the tank.
10. (a) Sketch the graph of $x^2 + 4y^2 - 2x + 16y + 1 = 0$, labeling the coordinates of the center, foci, vertices, and asymptotes, if any.
- (b) Find the area of the region that lies between the curves $y = 4 - x^2$ and $y = x$.
11. (a) Find the value of the following improper integral, or show it diverges:

$$\int_0^1 \frac{x^2}{x^3 - 1} dx$$

- (b) A curve is given parametrically by $x = t^2 - 2$, $y = t^3$, $-3 \leq t \leq 3$. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for $t \neq 0$, without eliminating the parameter t .

END OF EXAM